

Efficiently searching agent-state based policies in Dec-POMDPs

Aditya Mahajan
McGill University

Joint work with Amit Sinha and Matthieu Geist

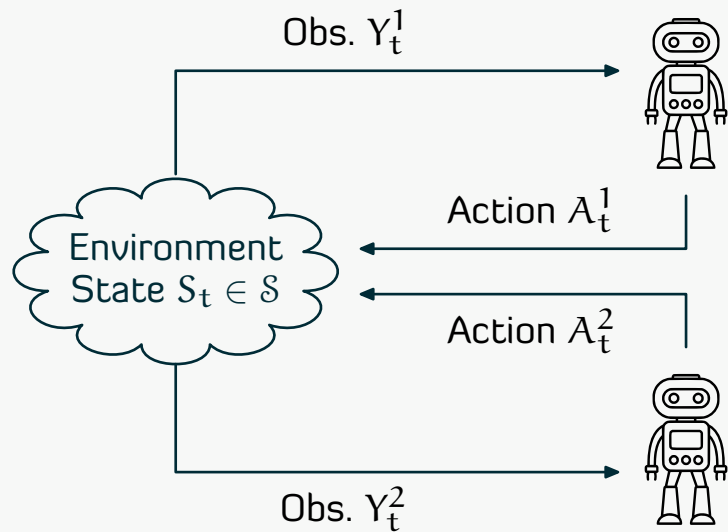
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21 July 2026

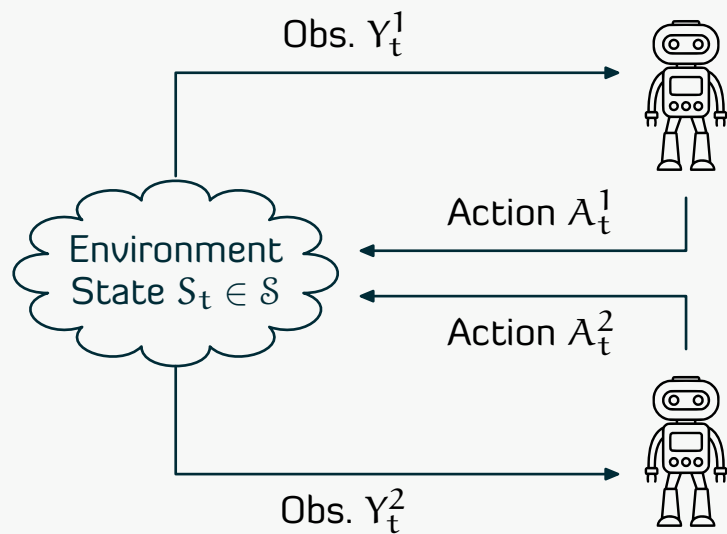
▶ email: aditya.mahajan@mcgill.ca

▶ web: <https://adityam.github.io>

Dec-POMDPs or multi-agent dynamic teams



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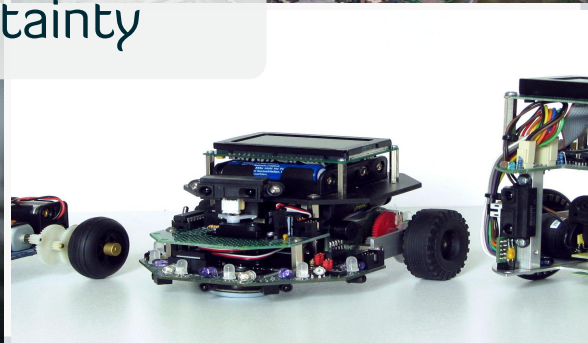
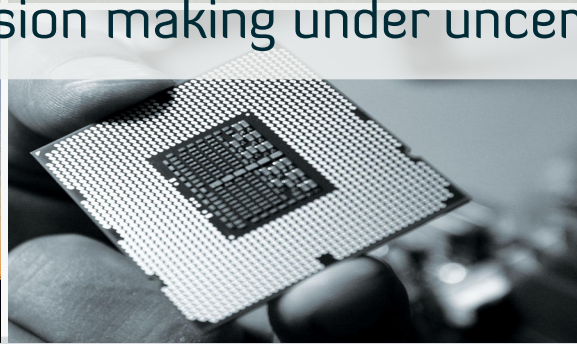
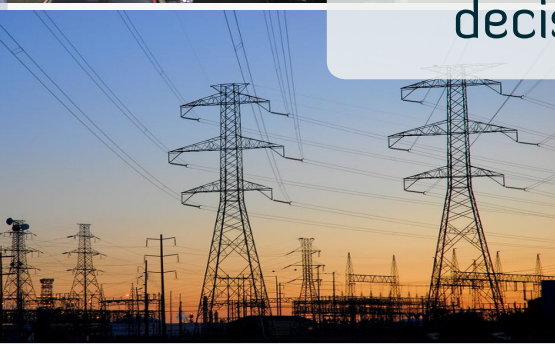


System model

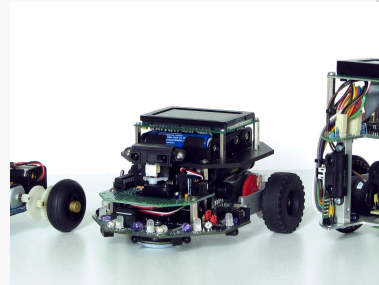
- ▶ Multiple agents
- ▶ Each sees only **local** observations
- ▶ must **coordinate** to maximize common reward
- ▶ **Key challenge:** Decentralized information



Common theme: multi-stage multi-agent
decision making under uncertainty

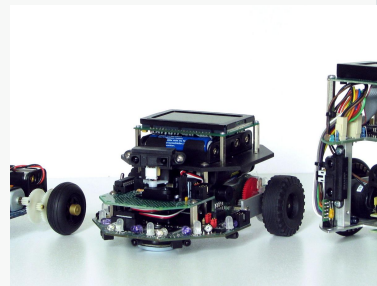


Networked control systems



Policy search for Dec-POMDPs—(Mahajan)

Networked control systems

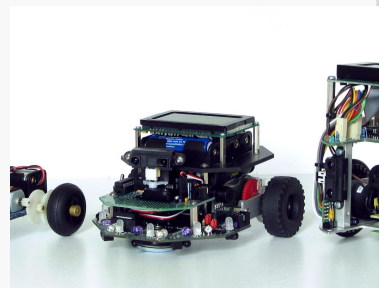


Networked control systems



Challenges

- Signals sent over wireless channels (packet drops)

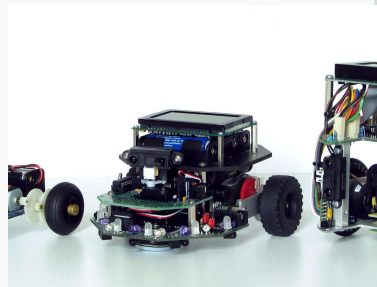


Networked control systems



Challenges

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- ▶ Different vehicles have different information

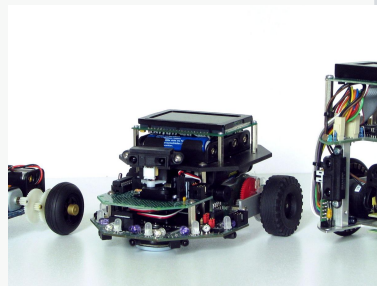


Networked control systems



Challenges

- ▶ Signals sent over wireless channels (packet drops)
- ▶ Different vehicles have different information
 - ▶ Decentralized control
 - ▶ Decentralized estimation
 - ▶ Decentralized learning



Salient Features of Modern Engineering Systems

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Multiple agents

Agents have different partial information about the environment and each other

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Decentralized Coordination

All agents must coordinate to achieve a system-wide objective

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Communication and Signaling

Possible to explicitly or implicitly communicate information



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Dynamics may not be completely known or may change over time

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Conceptual difficulties in Dec-POMDPs/team problems

Witsenhausen Counterexample

- ▶ A two-step dynamical system with two controllers
- ▶ Linear dynamics, quadratic cost, and Gaussian disturbance
- ▶ Non-linear controllers outperform linear control strategies . . .
. . . cannot use Kalman filtering + Riccati equations

 Witsenhausen, "A counterexample in stochastic optimum control," SICON 1968.

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Whittle and Rudge Example

- ▶ Infinite horizon dynamical system with two symmetric controllers
- ▶ Linear dynamics, quadratic cost, and Gaussian disturbance
- ▶ **A priori** restrict attention to linear controllers
- ▶ Best linear ctrls **don't** have finite dimensional representation

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📖 Whittle and Rudge, "The optimal linear solution of a symmetric team control problem," App. Prob. 1974.

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Complexity analysis

- ▶ All random variables are finite valued
- ▶ Finite horizon setup
- ▶ The problem of finding the best control strategy is in **NEXP**

📖 Witsenhausen, "A counterexample in stochastic optimum control," SICON 1968.

📖 Whittle and Rudge, "The optimal linear solution of a symmetric team control problem," App. Prob. 1974.

📖 Bernstein, et al., "The complexity of decentralized control of Markov decision processes," MOR 2002.

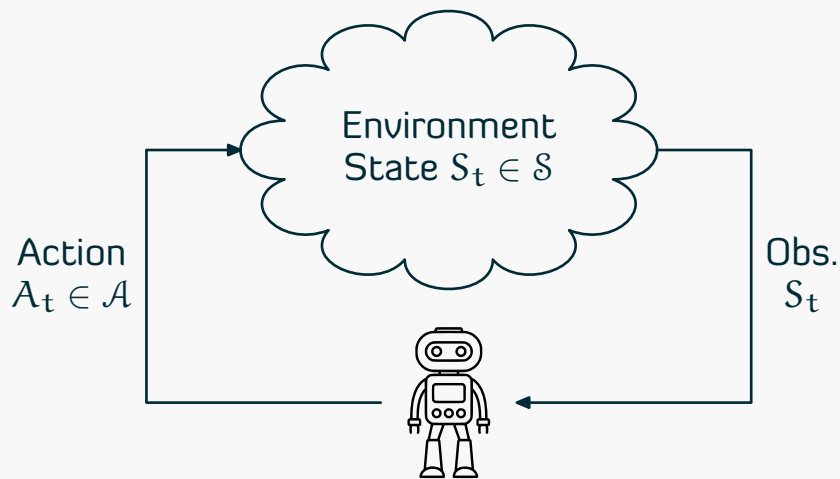
Why are multi-agent problems hard?

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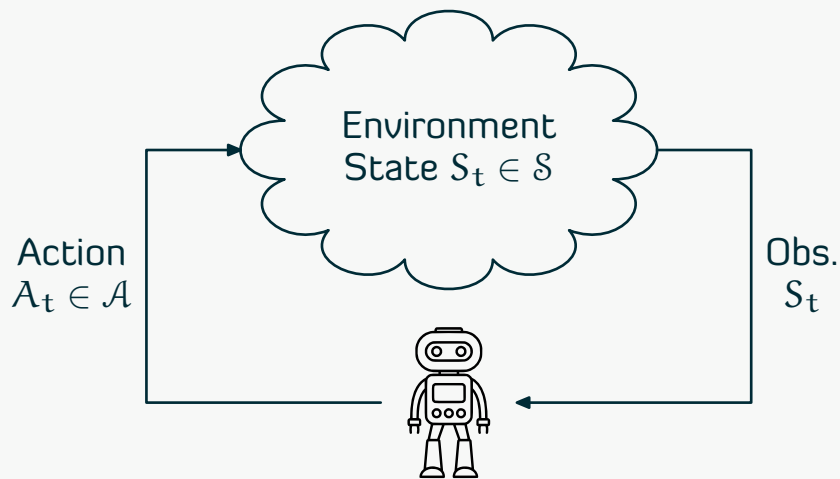
Why are single-agent problems easy?

Review: Markov decision processes (MDPs)

MDP: MARKOV DECISION PROCESS



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Dynamics: $\mathbb{P}(S_{t+1} | S_t, A_t)$

Observations: S_t

Reward $R_t = r(S_t, A_t)$.

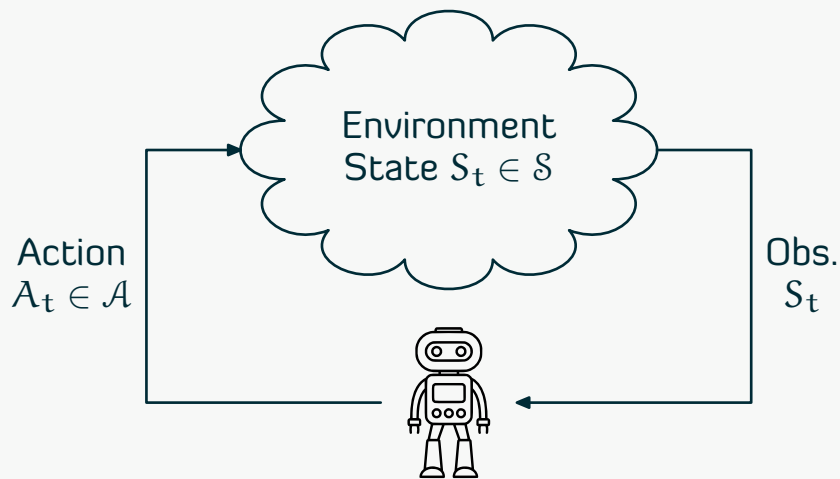
Action: $A_t \sim \pi_t(S_{1:t}, A_{1:t-1})$.

$\pi = (\pi_1, \dots, \pi_T)$ is called a **policy**.

Objective choose a policy π to maximize:

$$J(\pi) := \mathbb{E}^{\pi} \left[\sum_{t=1}^T R_t \right]$$

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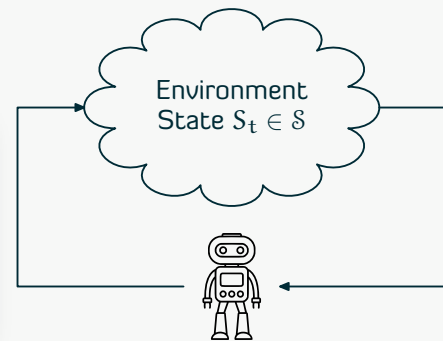
Conceptual challenge

- ▶ Brute force search has an exponential complexity in time horizon.
- ▶ How to efficiently search an optimal policy?

Review: Key simplifying ideas

Principle of Irrelevant information

No loss of optimality in choosing the action A_t as a function of the current state S_t

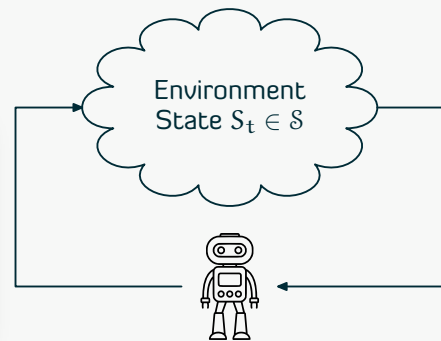


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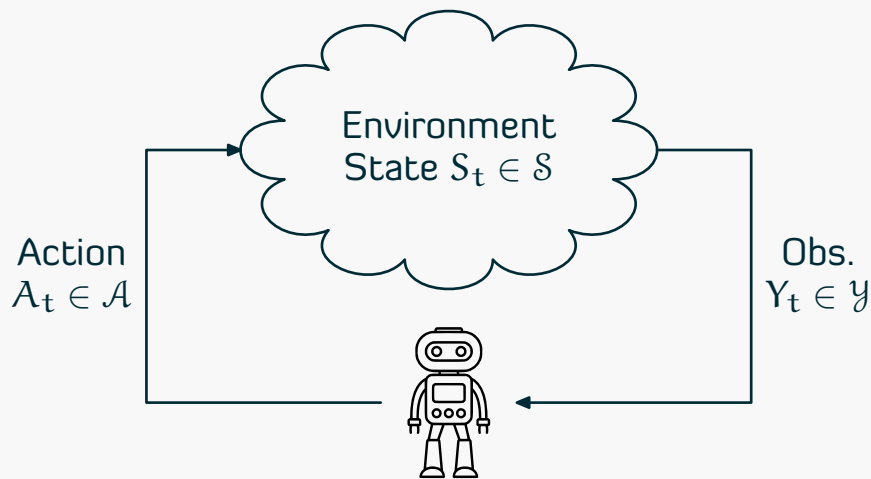
Principle of Optimality

The optimal control policy is given by a DP with state S_t :

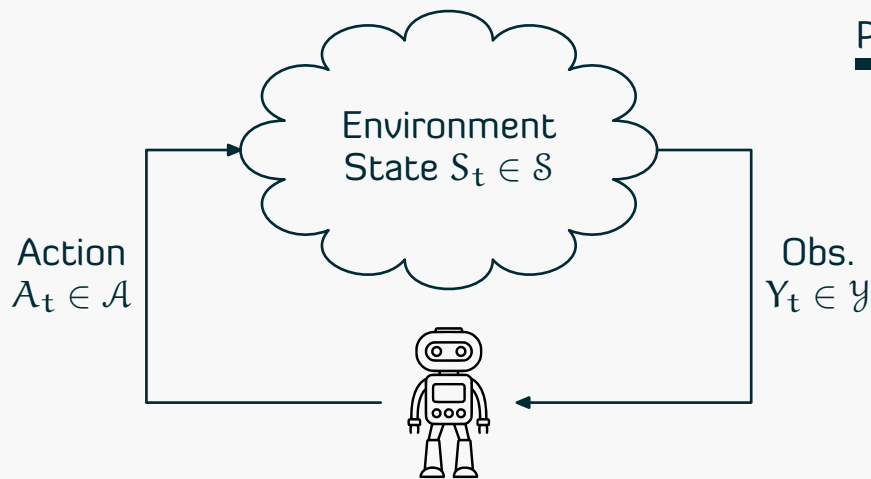
$$V_{T+1}(s) = 0, \quad V_t(s) = \max_{a \in \mathcal{A}} \left\{ r(s, a) + \int V_{t+1}(s') P(ds'|s, a) \right\}$$

Bellman, "Dynamic Programming," 1957.

Review: Partially observable MDPs (POMDPs)



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POMDP: PARTIALLY OBSERVABLE MDPS

$$\begin{aligned}\mathbb{P}(S_{t+1}, Y_{t+1} | S_{1:t}, Y_{1:t}, A_{1:t}) \\ = \mathbb{P}(S_{t+1}, Y_{t+1} | S_t, A_t)\end{aligned}$$

Reward: $R_t = r(S_t, A_t)$.

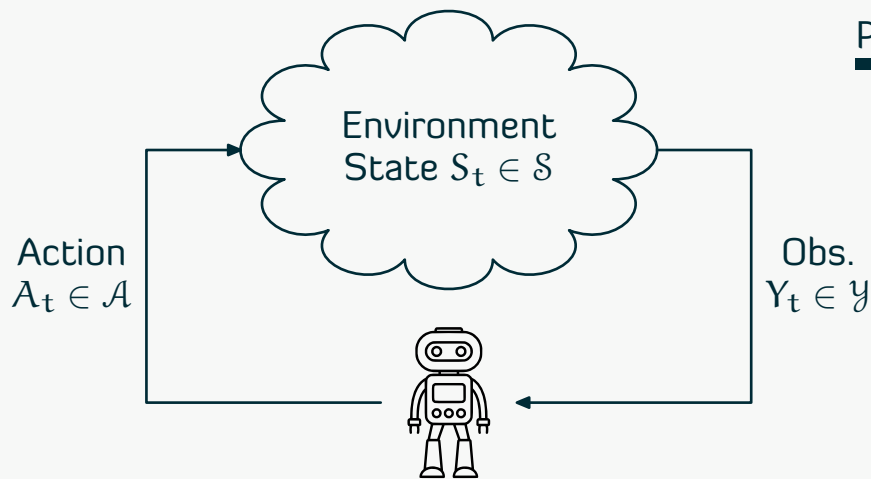
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Review: Belief-state based planning

Key simplifying idea

Define **belief state** $B_t \in \Delta(\mathcal{S})$ as $B_t(s) = \mathbb{P}(S_t = s \mid Y_{1:t}, A_{1:t-1})$.

► Belief state updates in a state-like manner: $B_{t+1} = \text{function}(B_t, Y_{t+1}, A_t)$.

► Belief state is sufficient to evaluate rewards: $\mathbb{E}[R_t \mid Y_{1:t}, A_{1:t}] = \hat{r}(B_t, A_t)$.

Thus, $\{B_t\}_{t=1}^T$ is a **perfectly observed** controlled Markov process.

📖 Astrom, "Optimal control of Markov processes with incomplete information," JMAA 1965.

📖 Stratonovich, "Conditional Markov Processes," TVP 1960.

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Thus, $\{B_t\}_{t=1}^T$ is a **perfectly observed** controlled Markov process. Therefore:

Structure of optimal policy

There is no loss of optimality in choosing the action A_t as a function of the belief state B_t

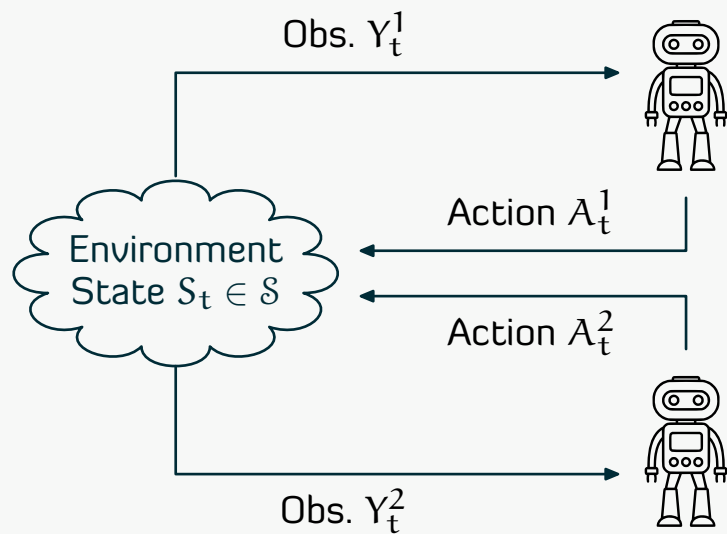
Dynamic Program

The optimal control policy is given by a DP with belief B_t as state.

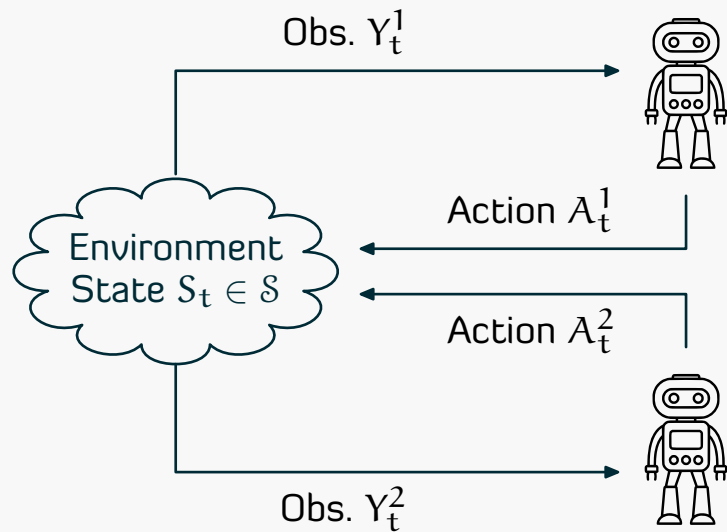
What happens in multi-agent settings?

Dec-POMDPs: Multi-agent partially observed control

Dec-POMDP: DECENTRALIZED POMDPS



Dec-POMDPs: Multi-agent partially observed control



Dec-POMDP: DECENTRALIZED POMDPS

$$\mathbb{P}(S_{t+1}, Y_{t+1}^1, Y_{t+1}^2 \mid S_{1:t}, Y_{1:t}^1, Y_{1:t}^2, A_{1:t}^1, A_{1:t}^2) \\ = \mathbb{P}(S_{t+1}, Y_{t+1}^1, Y_{t+1}^2 \mid S_t, A_t^1, A_t^2)$$

Obs: Y_t^i for each agent i (private histories H_t^i).

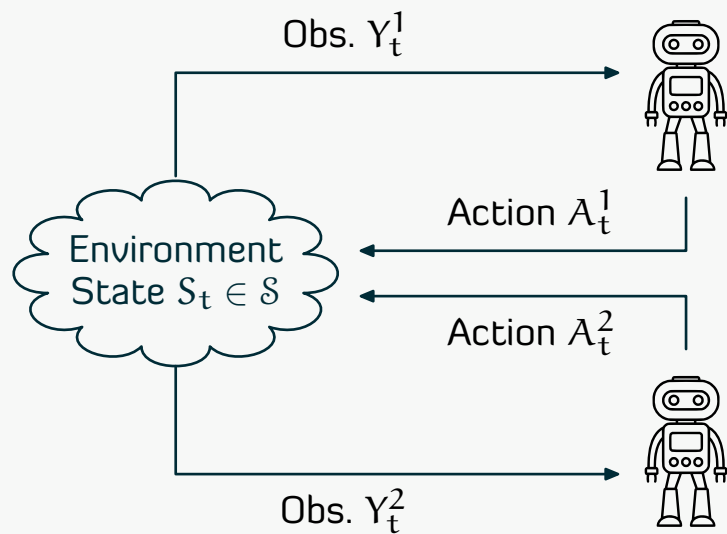
Team reward $R_t = r(S_t, A_t^1, A_t^2)$.

Action: $A_t^i \sim \pi_t^i(H_t^i)$, $H_t^i = (Y_{1:t}^i, A_{1:t-1}^i)$.
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Dec-POMDPs: Multi-agent partially observed control



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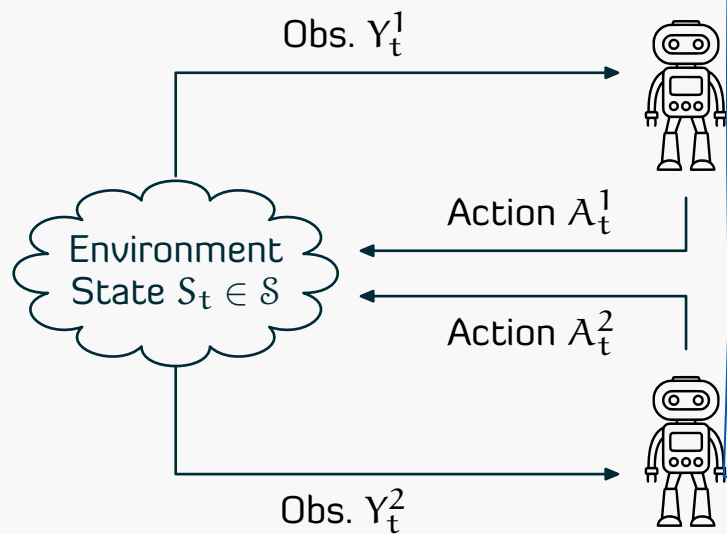
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- ▶ How to efficiently search for good policies?

Dec-POMDPs: Multi-agent partially observed control



Some remarks

- ▶ Simplest non-trivial setup for optimization over time with asymmetric information.
- ▶ no strategic considerations
(cf. non cooperative games)
- ▶ no splitting of rewards
(cf. cooperative games)

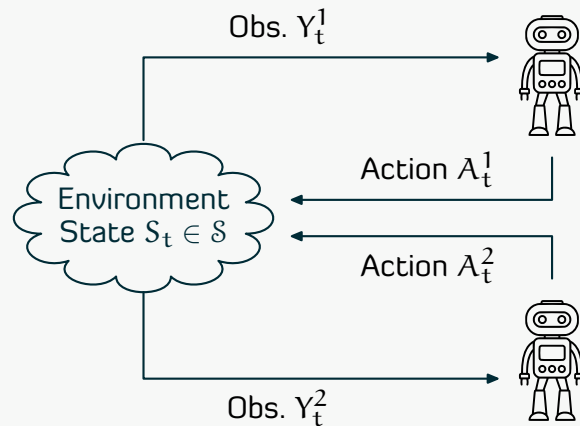
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Fundamental challenge in multi-agent systems

Strategic coupling

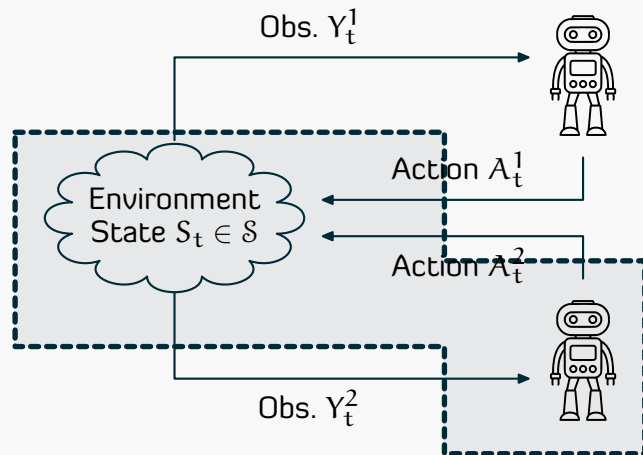
- ▶ The non-stationarity problem
- ▶ Second-guessing problem
- ▶ Signaling aspect of control



Fundamental challenge in multi-agent systems

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From the pov of agent 1

Sufficient statistic

- ▶ $b_t^{1,1} = \mathbb{P}(S_t, H_t^2 \mid H_t^1)$

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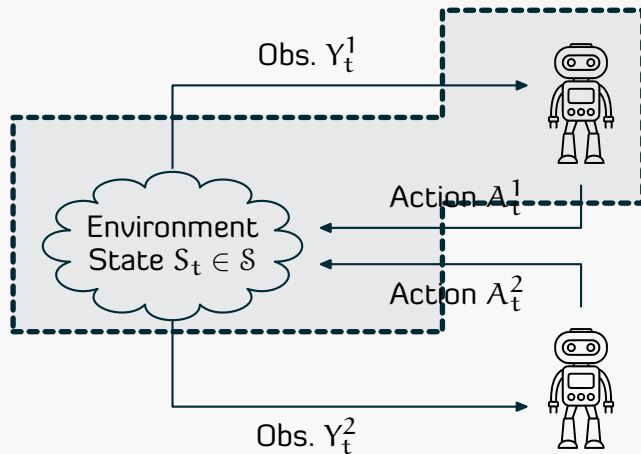
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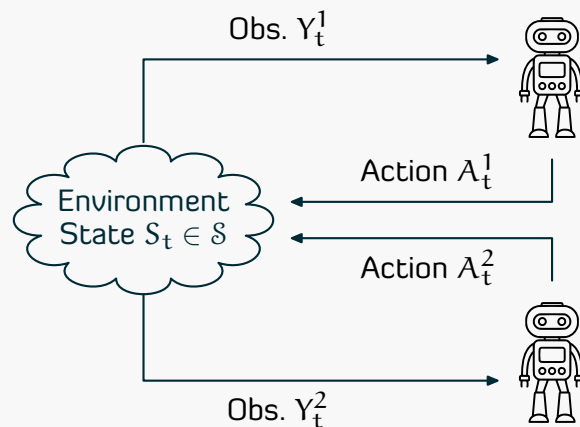
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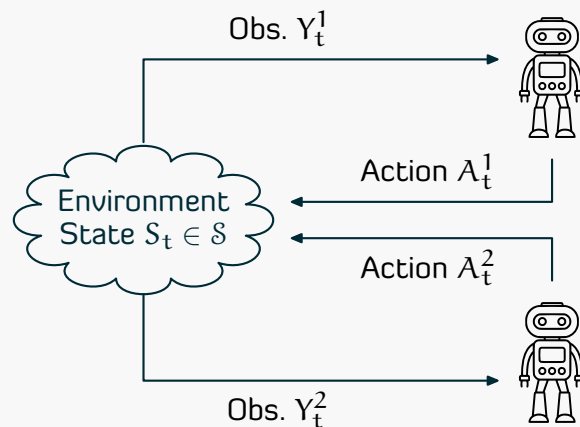
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- ▶ ...



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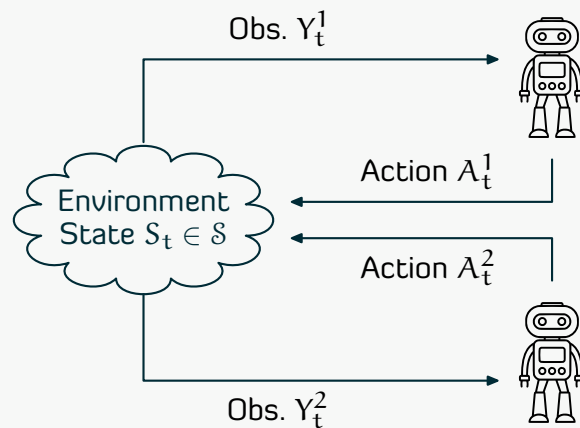
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Fundamental challenge in multi-agent systems

Strategic coupling

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Challenge

- ▶ An agent needs to form a belief on the belief of the belief of the ... other agent!
- ▶ Gives rise to a nested hierarchy of beliefs.
- ▶ How to find "joint" sufficient statistics?
- ▶ How to search for optimal policies?

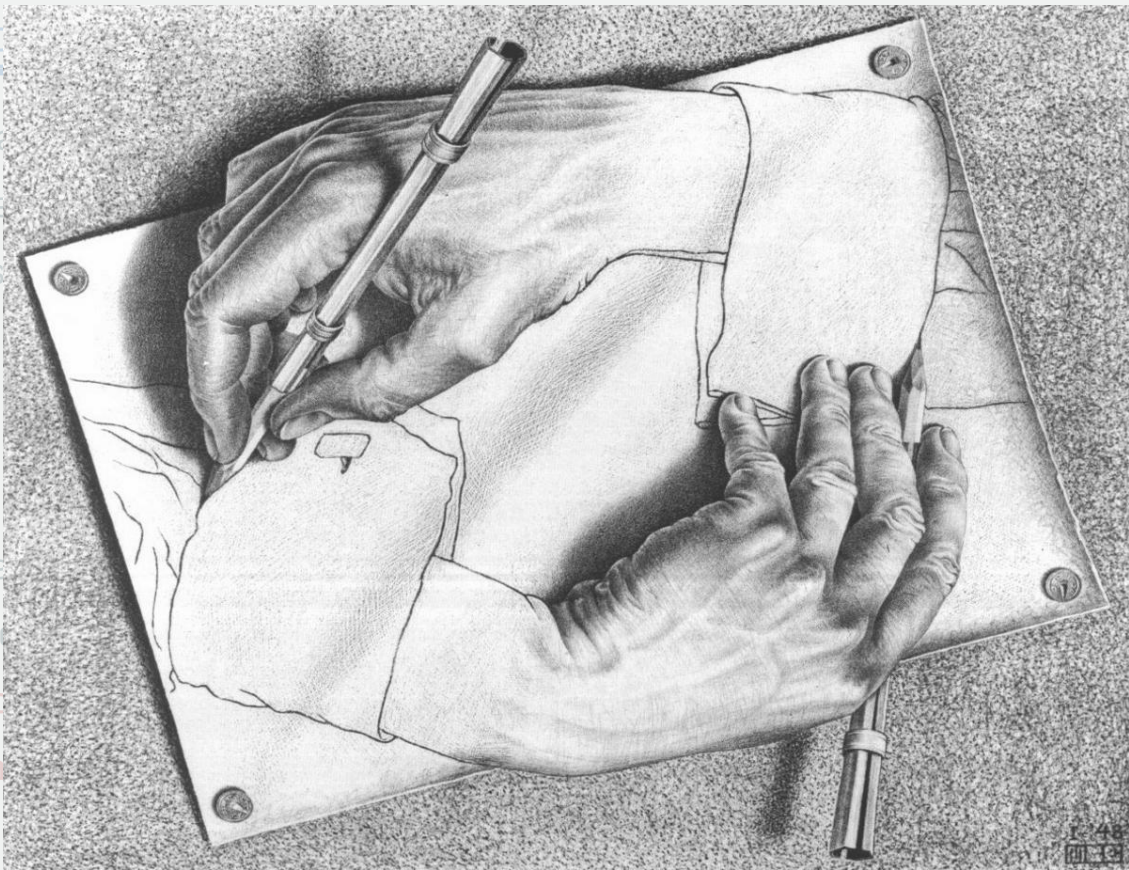
Fundamental challenge in multi-agent systems

Strategic cooperation

- ▶ The non-stationary environment
- ▶ Second-guessing other agents
- ▶ Signaling as a coordination mechanism

Challenge

- ▶ An agent needs to learn the intentions of other agents
- ▶ Gives rise to the problem of **coordination**
- ▶ **How to find a common goal**
- ▶ **How to search for a common solution**



It!

Revisit the modeling assumptions

Why assume that agents have perfect recall?

Why do we make this assumption?

- ▶ Perfect recall is an idealized assumption. No real system can have infinite memory.
- ▶ In single-agent systems (POMDPs), this assumption simplifies the analysis and leads to a policy that can be implemented with fixed memory.
- ▶ But this assumption leads to conceptual headaches in multi-agent systems!

Why assume that agents have perfect recall?

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- ▶ In single-agent systems (POMDPs), this assumption simplifies the analysis and leads to algorithms implemented with fixed memory.
- ▶ But this assumption leads to multi-agent systems!

Obvious fix: Don't make the assumption!

- ▶ Simply assume that all agents have fixed memory:
 - ▶ finite window of past observations
 - ▶ finite state machines
 - ▶ RNNs
- ▶ Similar assumption made in many recent RL papers
- ▶ **Key challenge:** decision process is no longer Markov!

How to search for optimal agent-state policies

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Searching for optimal policies

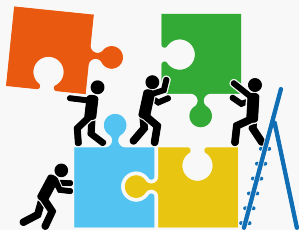
- ▶ Designer's approach
- ▶ Simple idea, hard to scale

How to search for optimal agent-state policies



Searching for optimal policies

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Searching for approximately optimal policies

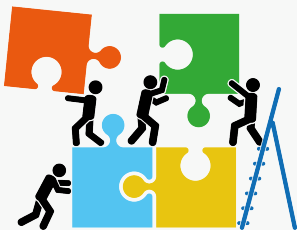
- ▶ Approximate information state
- ▶ Works for POMDPs, doesn't work for Dec-POMDPs

How to search for optimal agent-state policies



Searching for optimal policies

- ▶ Designer's approach
- ▶ Simple idea, hard to scale



Searching for approximately optimal policies

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Searching for team sequential equilibrium

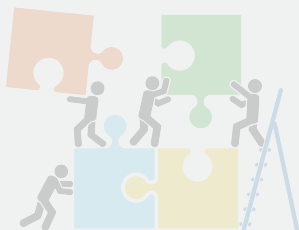
- ▶ ... with some additional ingredients
- ▶ Works remarkably well for POMDPs and Dec-POMDPs

How to search for optimal agent-state policies



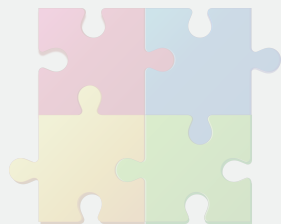
Searching for optimal policies

- ▷ Designer's approach
- ▷ Simple idea, hard to scale



Searching for approximately optimal policies

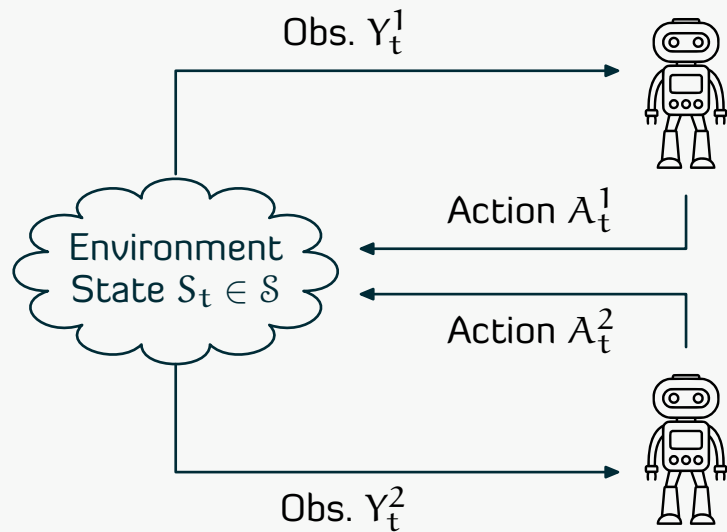
- ▷ Approximate information state
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Searching for team sequential equilibrium

- ▷ ... with some additional ingredients
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Dec-POMDPs with agent-state based policies

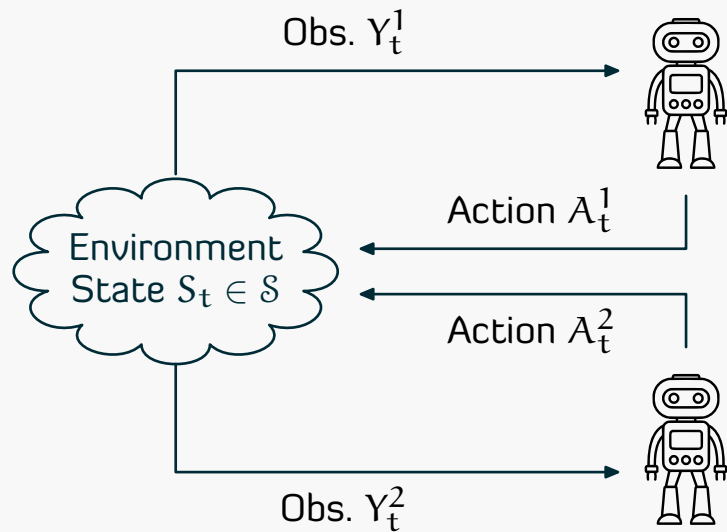


Agent state: $Z_t^i \in \mathcal{Z}^i$, where

$$(A_t^i, Z_{t+1}^i) = \pi_t^i(Y_t^i, Z_t^i)$$

- ▶ Can fix agent-state update function, e.g.,
 $Z_t^i = (Y_{t-k:t}^i, A_{t-k:t-1}^i)$
- ▶ Agent state may be real-valued (e.g., RNNs)

Dec-POMDPs with agent-state based policies



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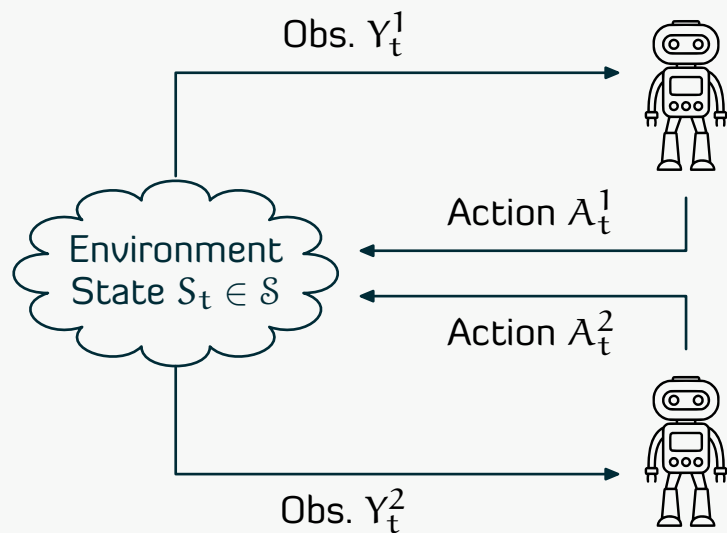
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Objective choose joint (π^1, π^2) to maximize:

$$J(\pi^1, \pi^2) := \mathbb{E}^{\pi^1, \pi^2} \left[\sum_{t=1}^T R_t \right]$$

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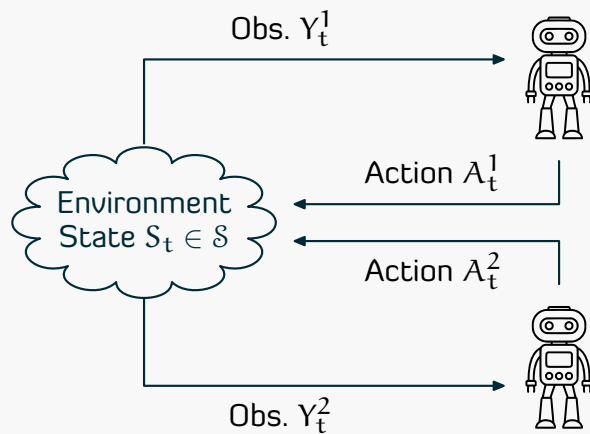
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Conceptual challenge

- ▶ Brute force search has an exponential complexity in time horizon and number of agents.
- ▶ How to efficiently search for good agent-state policies?

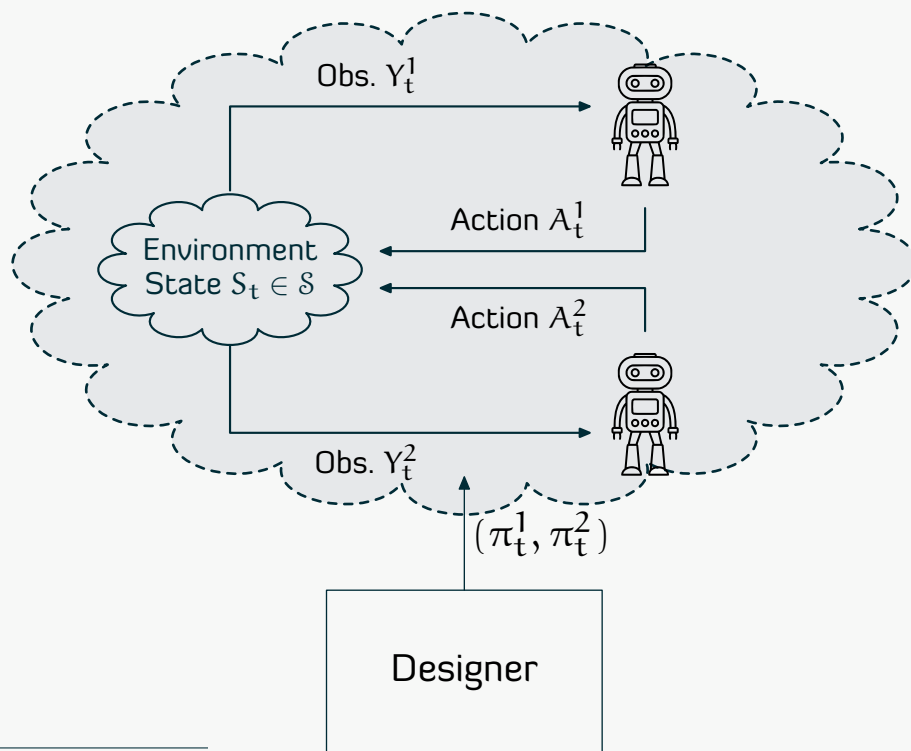
Designer's approach to find optimal policy



Witsenhausen, "A standard form for sequential stochastic control," Math. Systems Theory, 1973.

Mahajan, "Sequential decomposition of sequential teams", PhD thesis, 2008.

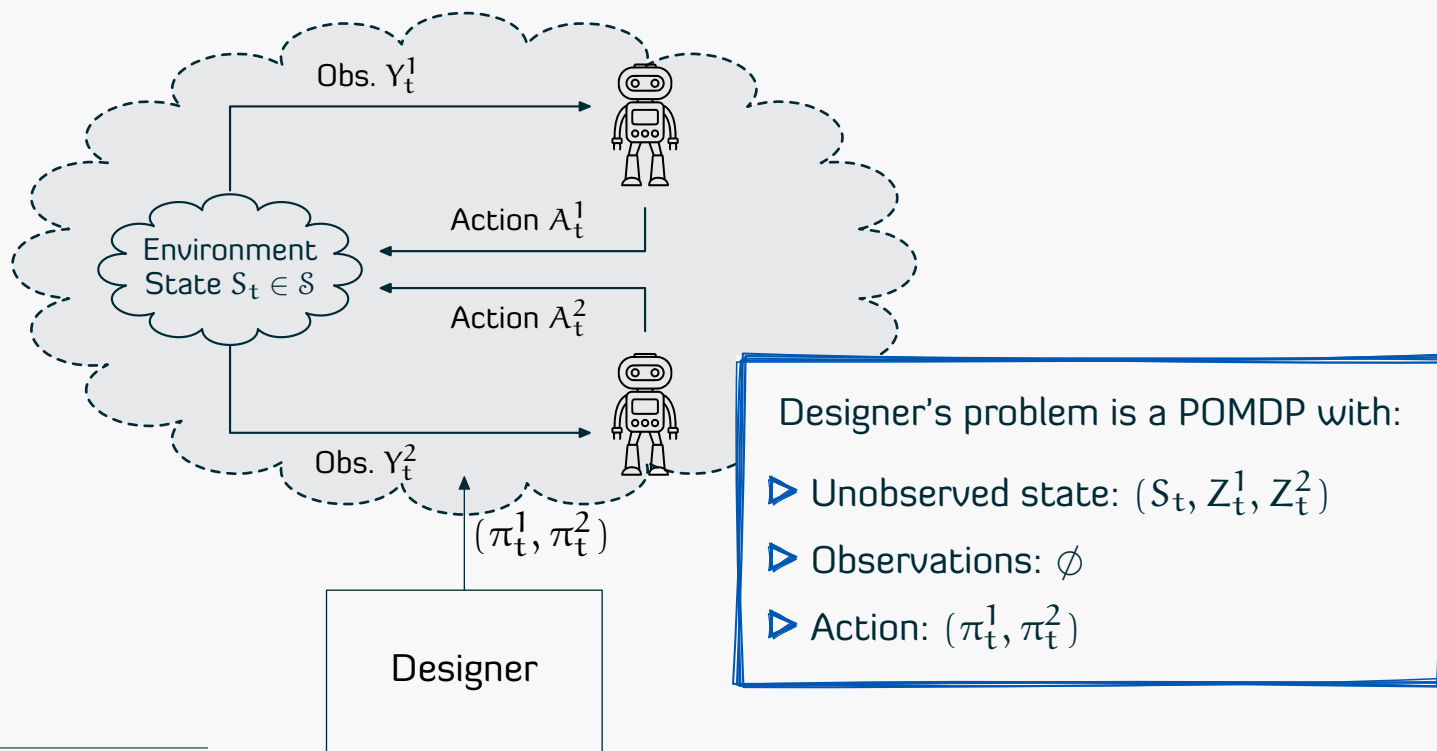
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Designer's approach to find optimal policy

Joint distribution
of env and
agent states

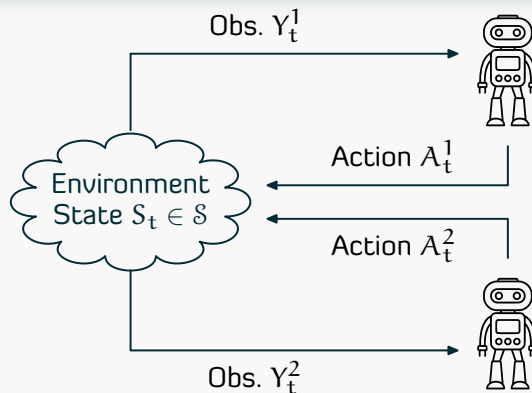
For any $\pi = (\pi^1, \pi^2)$, define

$$\xi_t^\pi(s, z^1, z^2) := \mathbb{P}^\pi(S_t = s, Z_t^1 = z^1, Z_t^2 = z^2)$$

Then:

$$\triangleright \xi_{t+1}^\pi = \phi_{\text{DES}}(\xi_t^\pi, \pi_t).$$

$$\triangleright \mathbb{E}^\pi[R_t] = r_{\text{DES}}(\xi_t^\pi, \pi_t).$$



Designer's approach to find optimal policy

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DP using
designer's
approach

Consider the following DP:

$$V_{\text{DES}, t}(\xi) = \max_{\pi} \{ r_{\text{DES}}(\pi, \xi) + V_{\text{DES}, t+1}(\phi_{\text{DES}}(\pi, \xi)) \}.$$

Let $\psi_{\text{DES}, t}(\xi)$ denote any arg max of the RHS. Let $\xi_1^* = \xi_1$ and recursively define

$$\pi_t^* = \psi_{\text{DES}, t}(\xi_t^*) \quad \text{and} \quad \xi_{t+1}^* = \phi_{\text{DES}, t}(\pi_t^*, \xi_t^*).$$

Then, the policy $\pi^* = (\pi_1^*, \dots, \pi_T^*)$ is optimal.

Some comments

Historical review

- ▶ The idea goes back to Witsenhausen's standard form (1973).
- ▶ Used for POMDPs in Sandell (1974) and general finite state Dec-POMDPs in Mahajan (2008).
- ▶ Related to Observational MDP approach of Dibangoye et al (2016).

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Limitations

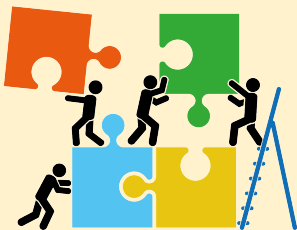
- ▶ The action space of the DP is the space of all policies (π_t^1, π_t^2) .
- ▶ Difficult to scale using standard sampling-based methods.

How to search for optimal agent-state policies



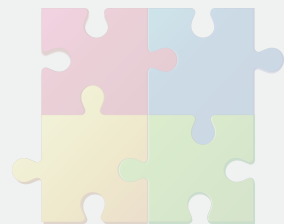
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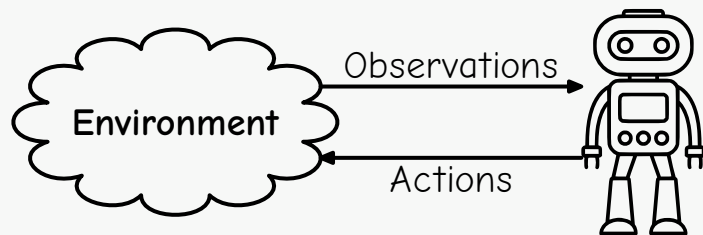
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Simplify: Consider POMDPs with agent-state policies



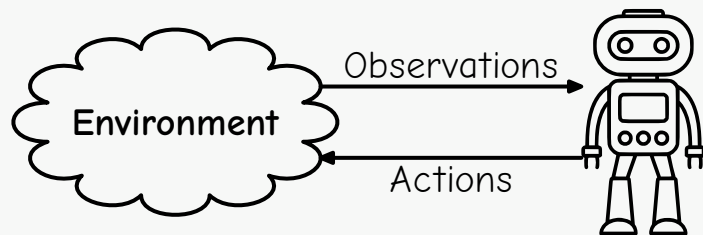
Further simplification: Fix agent state update

$$Z_{t+1} = \varphi_t(Z_t, Y_{t+1}, A_t)$$

By unrolling this assumption, we can say

$$Z_t = \sigma_t(H_t)$$

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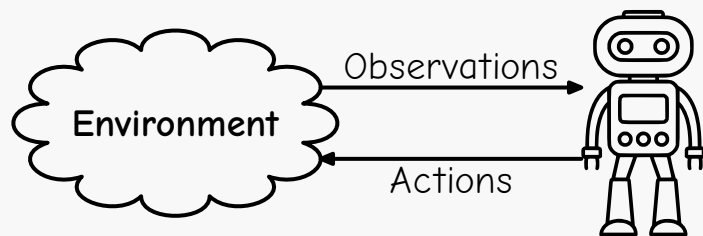
Approximate information state (AIS)

$\{Z_t\}_{t=1}^T$ is an (ε, δ) -AIS, where $\varepsilon = (\varepsilon_1, \dots, \varepsilon_T)$ and $\delta = (\delta_1, \dots, \delta_T)$, if there exist

$$\underbrace{r_{\text{AIS}, t}: \mathcal{Z} \times \mathcal{A} \rightarrow \mathbb{R}}_{\text{approx reward estimator}} \quad \text{and} \quad \underbrace{P_{\text{AIS}, t}: \mathcal{Z} \times \mathcal{A} \rightarrow \Delta(\mathcal{Z})}_{\text{approx dynamic pred}}$$

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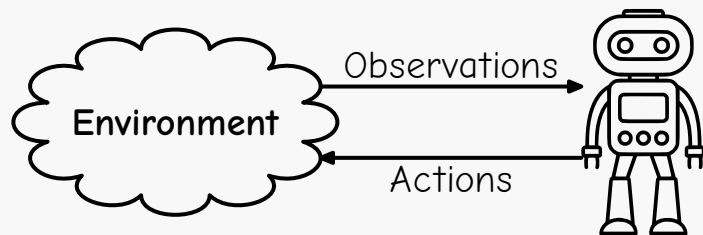
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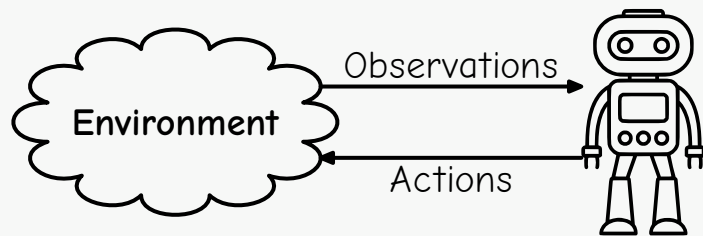
- ▶ reward est. is approx sufficient stat : $|\mathbb{E}[R_t \mid h_t, a_t] - r_{\text{AIS}, t}(\sigma_t(h_t), a_t)| \leq \varepsilon_t$
- ▶ dynamics are approx self predictive : $d(\mathbb{P}(Z_{t+1} \mid h_t, a_t), P_{\text{AIS}, t}(Z_{t+1} \mid \sigma_t(h_t), a_t)) \leq \delta_t$

Approx solutions for POMDPs with agent-state policies



- ▶ The pair (P_{AIS}, r_{AIS}) defines an MDP.
- ▶ Let π_{AIS} be the optimal policy of this MDP
- ▶ Then $\pi_t(h_t) = \pi_{AIS,t}(\sigma_t(h_t))$ is a feasible agent-state policy.

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Main result

- ▶ The agent-state policy π is α -optimal, where α depends on (ϵ, δ) and properties of V_{AIS} (depending on the choice of metric d)
- ▶ Can use $\lambda\epsilon^2 + (1 - \lambda)\delta^2$ as an auxiliary loss for RL algorithms.
- ▶ Works beautifully for Q-learning and actor-critic algorithms

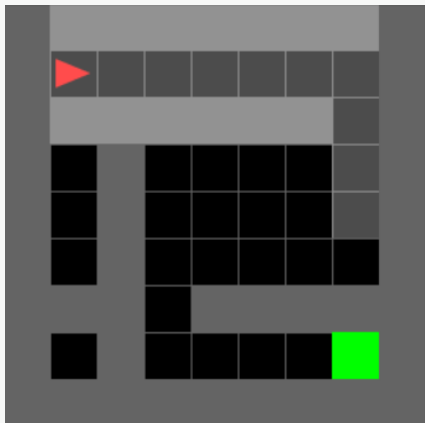
📖 Subramanian, Sinha, Seraj, and Mahajan, "Approximate information state for . . . partially observed systems", JMLR 2022.

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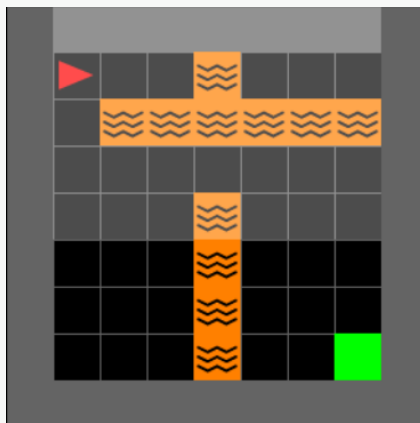
📖 Sinha, Geist, Mahajan, "Periodic agent-state based Q-learning for POMDPs", NeurIPS 2024.

Policy search for Dec-POMDPs—(Mahajan)

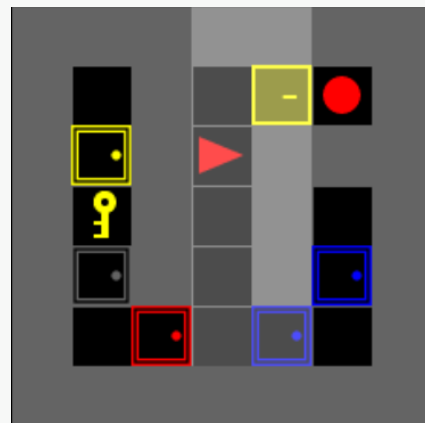
MiniGrid Environments



Simple Crossing

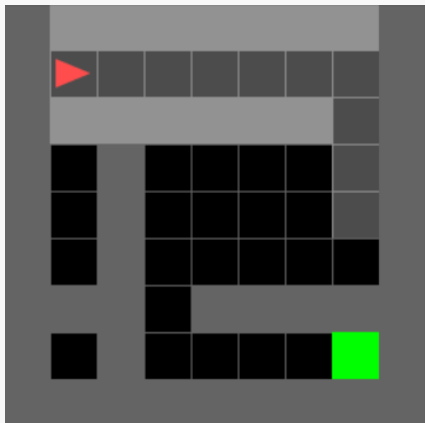


Lava Crossing

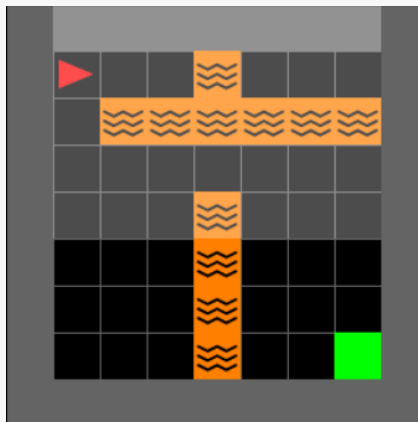


Key Corridor

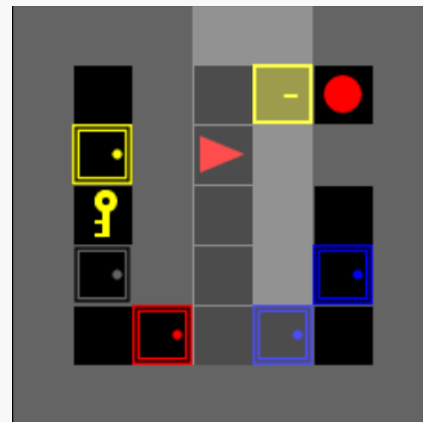
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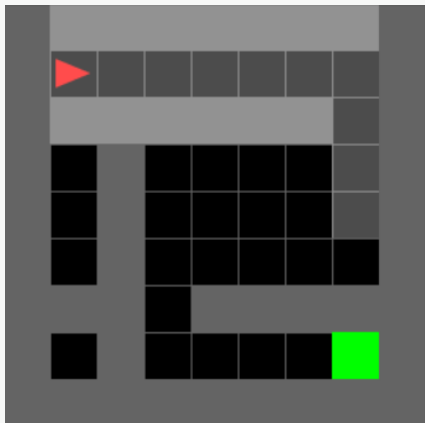


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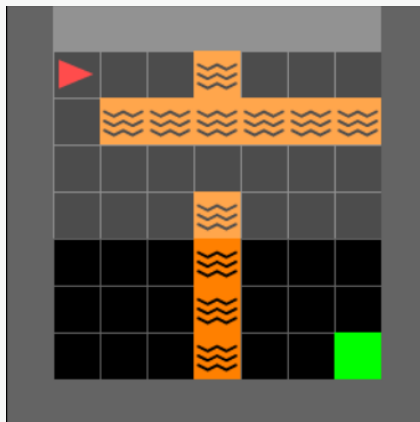
Features

- ▶ Partially observable 2D grids. Agent has a view of a 7×7 field in front of it. Observations are obstructed by walls.
- ▶ Multiple entities (agents, walls, lava, boxes, doors, and keys)
- ▶ Multiple actions (Move Forward, Turn Left, Turn Right, Open Door/Box, ...)

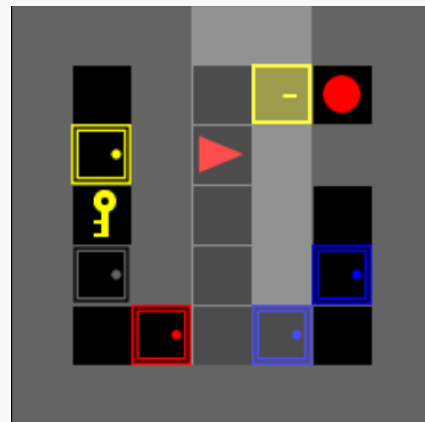
MiniGrid Environments



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Key Corridor

Algorithms

AIS + MMD

AIS with MMD as IPM

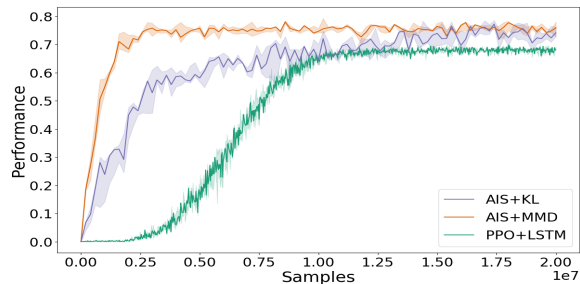
AIS + KL

AIS with KL as upper bound
of Wasserstein distance

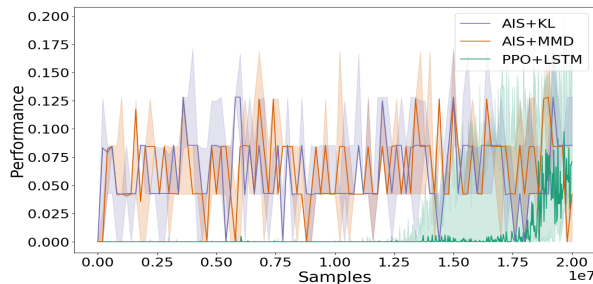
PPO + LSTM

Baseline proposed in paper
introducing minigrid envs

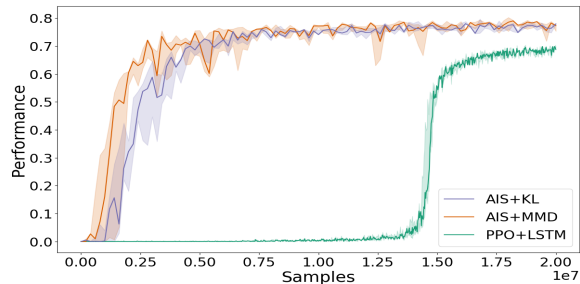
Significant improvement over baseline policies



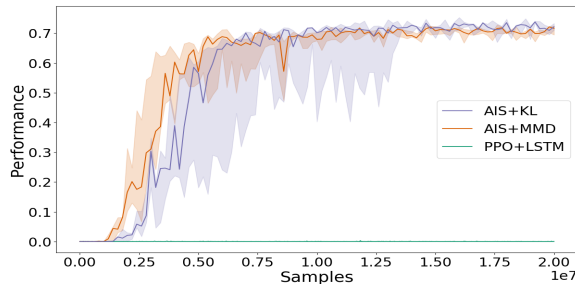
Simple Crossing S9N3



Lava Crossing S9N2



Doorkey 8x8

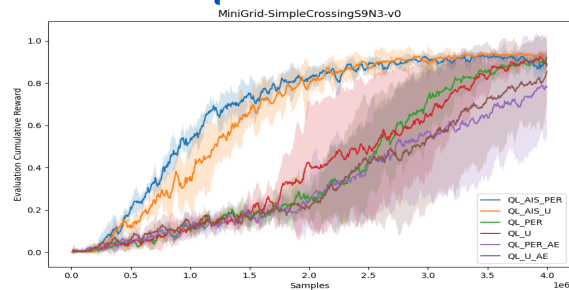


Key Corridor S3R2

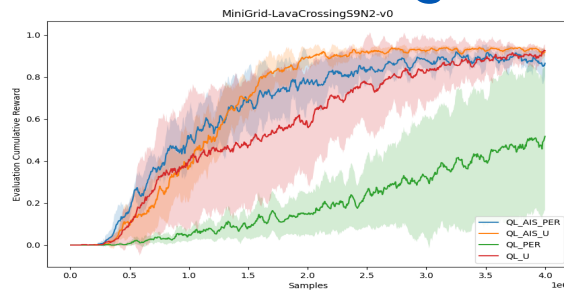
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Policy search for Dec-POMDPs—(Mahajan)

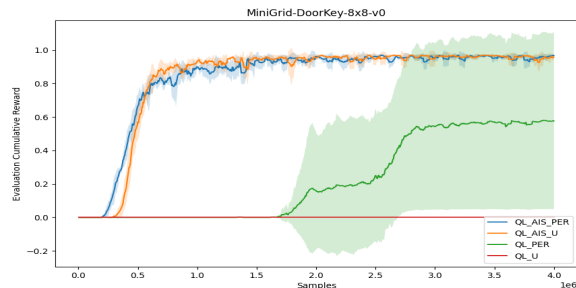
Similar improvements for AIS-enhanced Q-learning



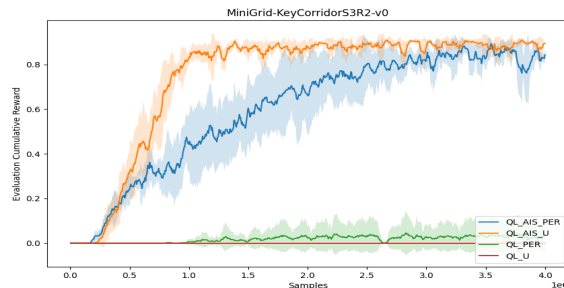
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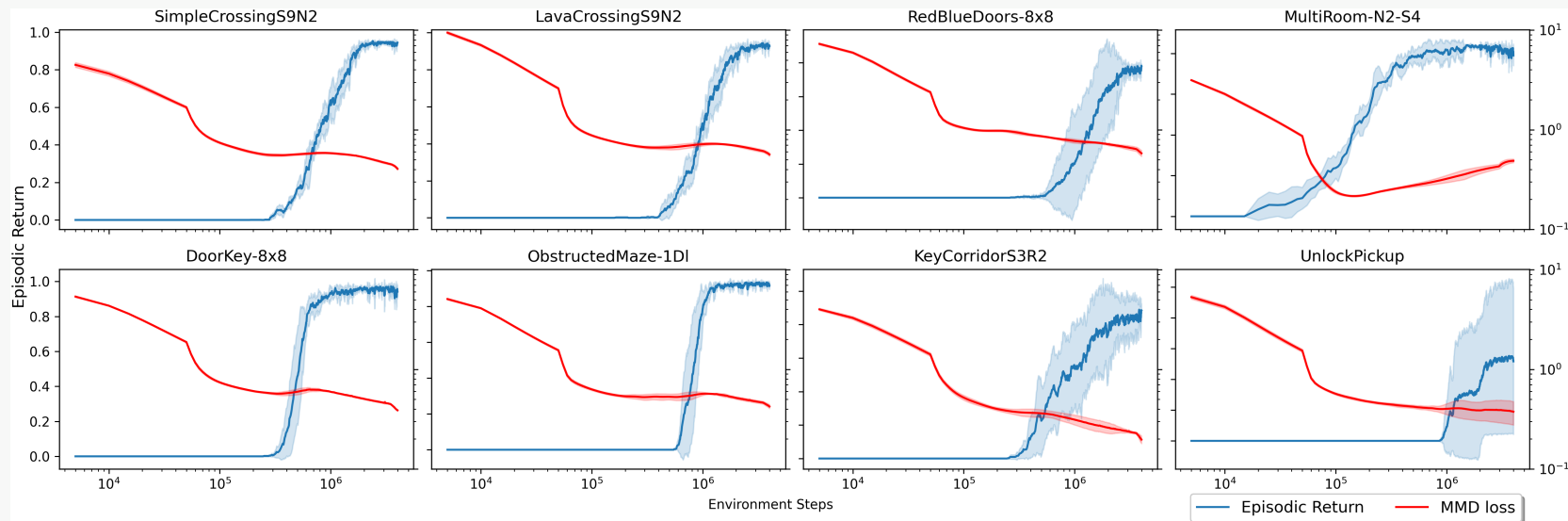
Key Corridor S3R2

📖 Ni, et al., "Bridging State and History Representations: Understanding self-predictive RL", ICLR 2024.

📖 Sinha, Geist, Mahajan, "Periodic agent-state based Q-learning for POMDPs", NeurIPS 2024.

Policy search for Dec-POMDPs—(Mahajan)

AIS loss vs. performance



Sayedsalehi , Akbarzadeh, Sinha , and Mahajan, "AIS based convergence analysis of recurrent Q-learning", EWRL 2023.

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How do we extend the notion
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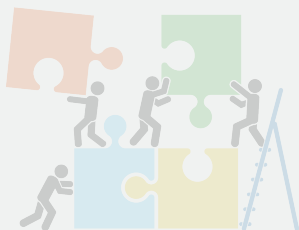
Some theoretical results using common-
information approach, but difficult to
scale to solve common benchmarks.

How to search for optimal agent-state policies



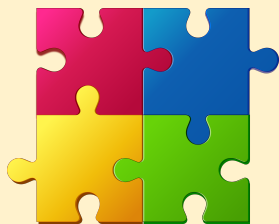
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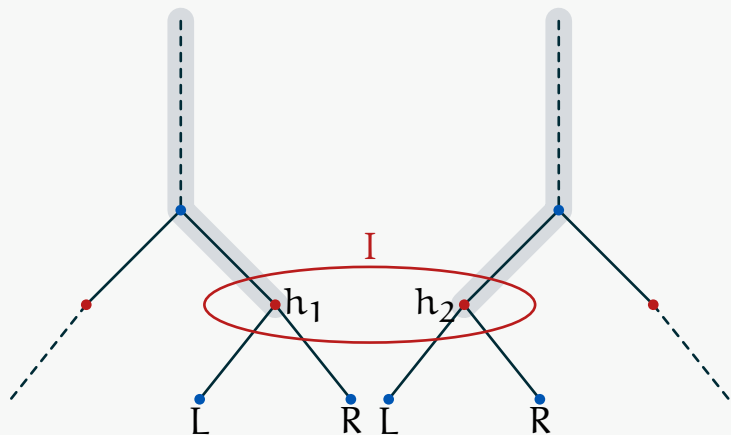
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Searching for team sequential equilibrium

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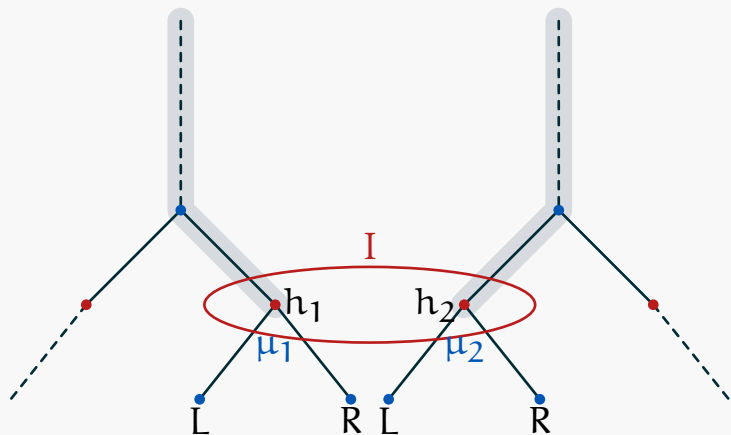
Dealing with information decentralization in Game Theory



▷ $Q(I, L) = ?$

▷ $Q(I, R) = ?$

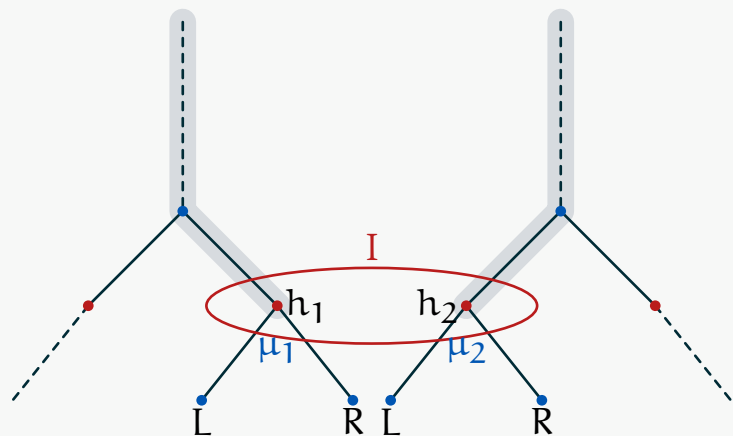
Dealing with information decentralization in Game Theory



$$\triangleright Q(I, L) = \mu_1 u(h_1, L) + \mu_2 u(h_2, L)$$

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Dealing with information decentralization in Game Theory



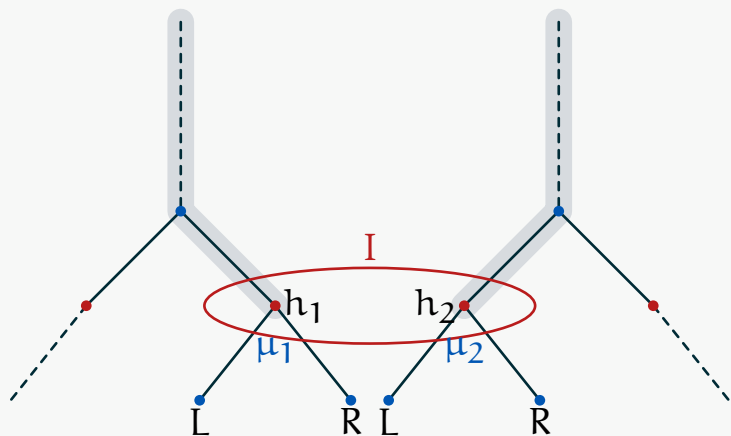
Games with asymmetric information

- ▶ Equilibrium is no longer just a strategy profile
- ▶ (strategy profile, **belief system**)
- ▶ Beliefs need to be consistent with strategy
- ▶ Strategies are best response to beliefs, where agents average at info sets using beliefs

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Dealing with information decentralization in Game Theory



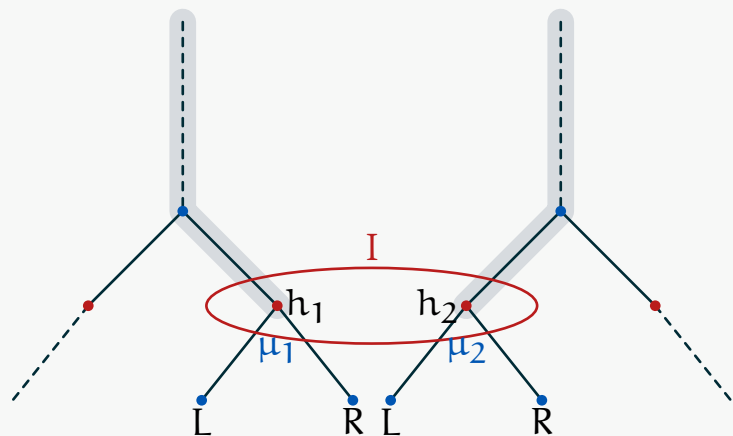
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- ▶ Perfect Bayesian Equilibrium
- ▶ Sequential Equilibrium
- ▶ ...

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Solution concepts

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- ▶ ...

Omitting several details

- ▶ Beliefs are updated according to Bayes' rule
- ▶ It is assumed that agents must have perfect recall
- ▶ Need delicate limit arguments when info sets have zero probability

Team sequential equilibrium for POMDPs

Team sequential equilibrium

A team sequential equilibrium is a collection of

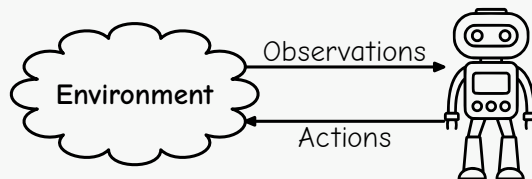
▶ a policy $\pi = (\pi_1, \dots, \pi_T)$

▶ a belief system $\xi = (\xi_1, \dots, \xi_T)$

such that

▶ **Sequential rationality** The policy π_t is best response when averaged using beliefs.

▶ **Consistency** **How do we define consistency?**



What does Bayesian update mean for a model without perfect recall?

Team sequential equilibrium for POMDPs

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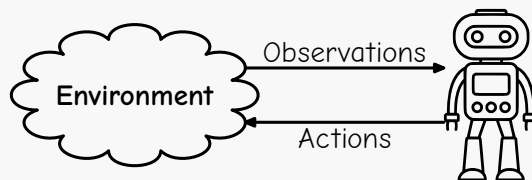
▶ a policy $\pi = (\pi_1, \dots, \pi_T)$

▶ a belief system $\xi = (\xi_1, \dots, \xi_T)$

such that

▶ **Sequential rationality** The policy π_t is best response when averaged using beliefs.

▶ **Consistency** **How do we define consistency?**



What does Bayesian update mean for a model without perfect recall?

Main idea: Track unconditional “beliefs”

How to find team sequential equilibrium for POMDPs

Notation

▶ $X_t = (S_t, Y_t, Z_t)$

▶ $U_t = (A_t, Z_{t+1})$

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Observation

- ▶ $\{X_t\}_{t \geq 1}$ is a controlled Markov process, controlled by $\{U_t\}_{t \geq 1}$
- ▶ However, $\pi_t: X_t \mapsto U_t$ is **not a valid policy**
- ▶ $\pi_t: (Y_t, Z_t) \mapsto U_t$ can be evaluated using standard formulas

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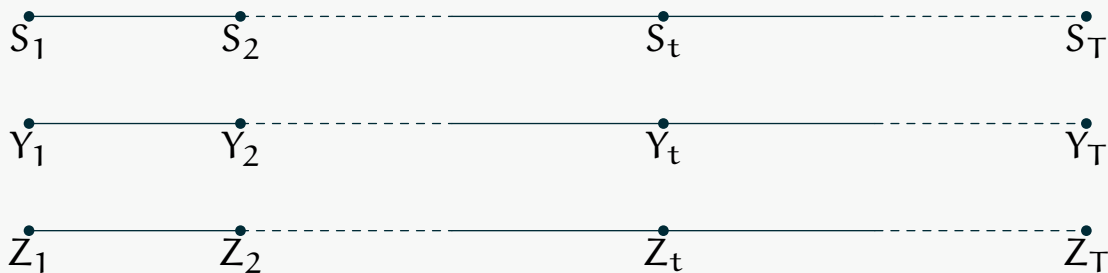
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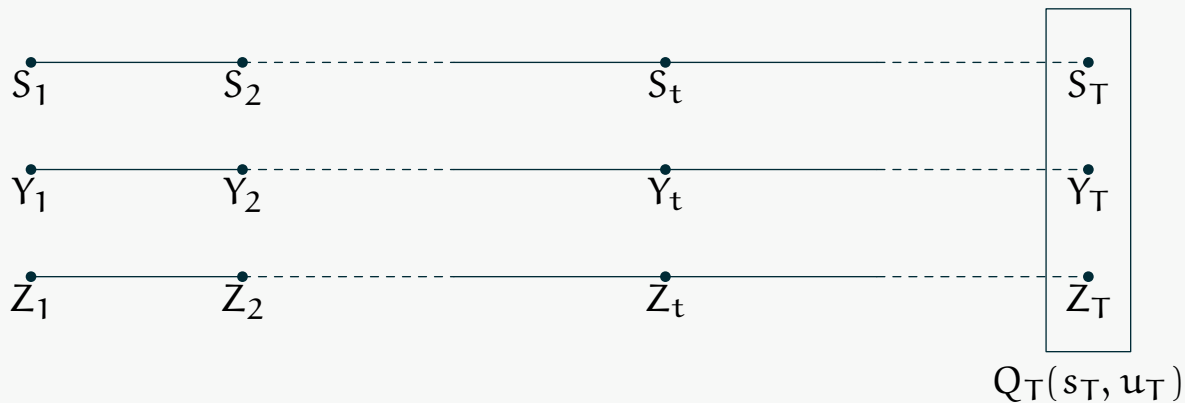
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▶ $X_t = (S_t, Y_t, Z_t)$

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Step 1: Backward pass

▶ Compute policy evaluation Q functions $Q_t(x_t, u_t)$



How to find team sequential equilibrium for POMDPs

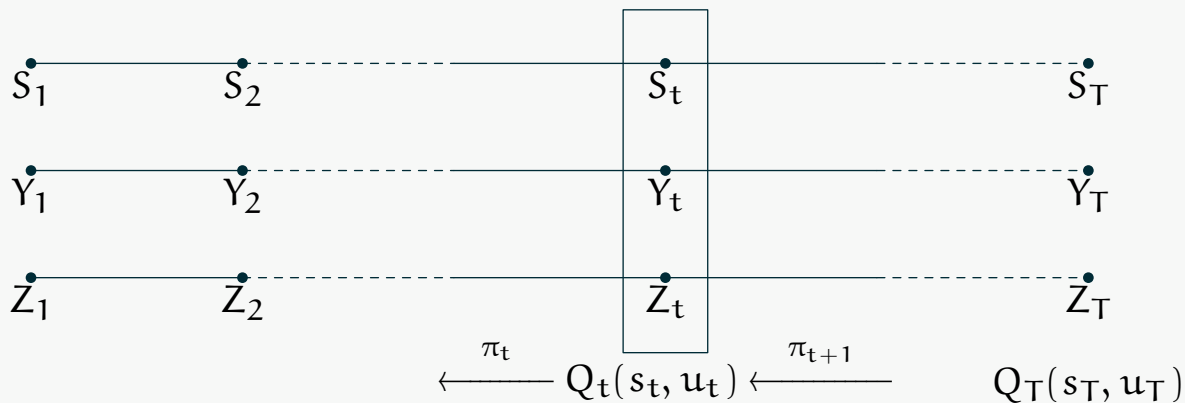
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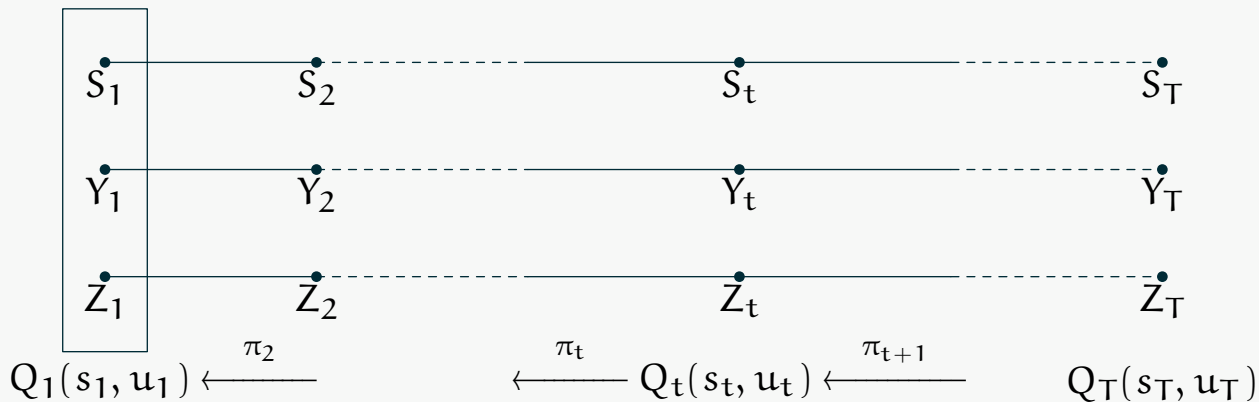
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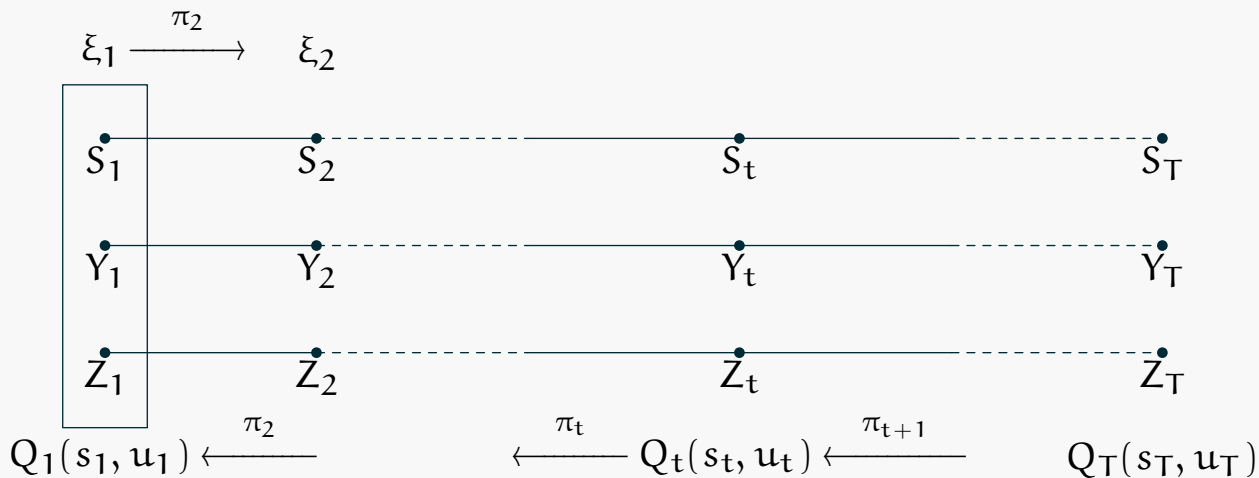
▷ $X_t = (S_t, Y_t, Z_t)$

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Step 2: Forward pass

▷ Compute marginal distribution $\xi_t(x)$ of Markov chain $\{X_t\}_{t \geq 1}$

▷ Use $\xi_t(s_t|y_t, z_t)$ to denote conditional beliefs wrt ξ_t



How to find team sequential equilibrium for POMDPs

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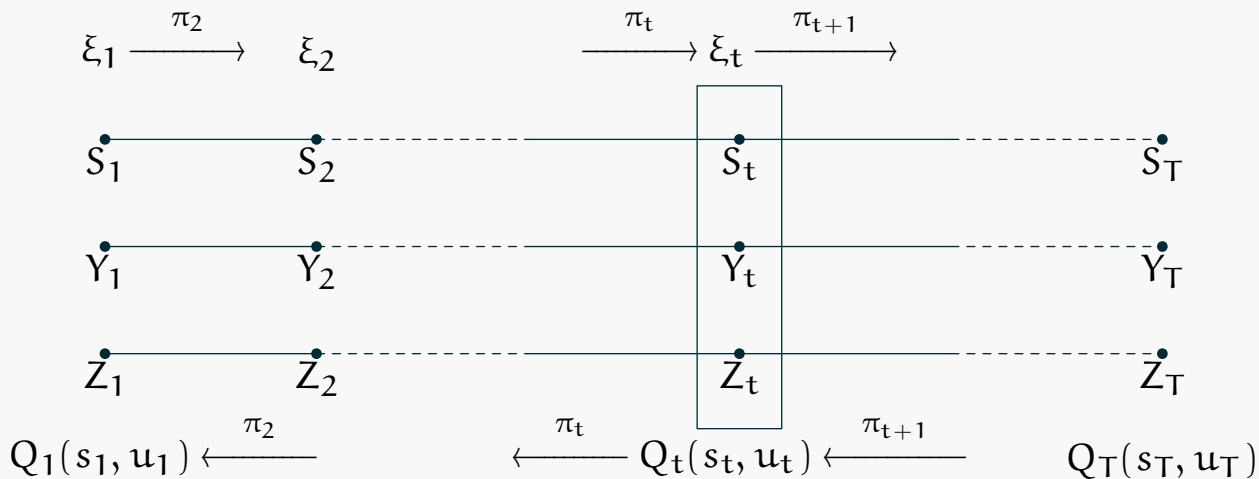
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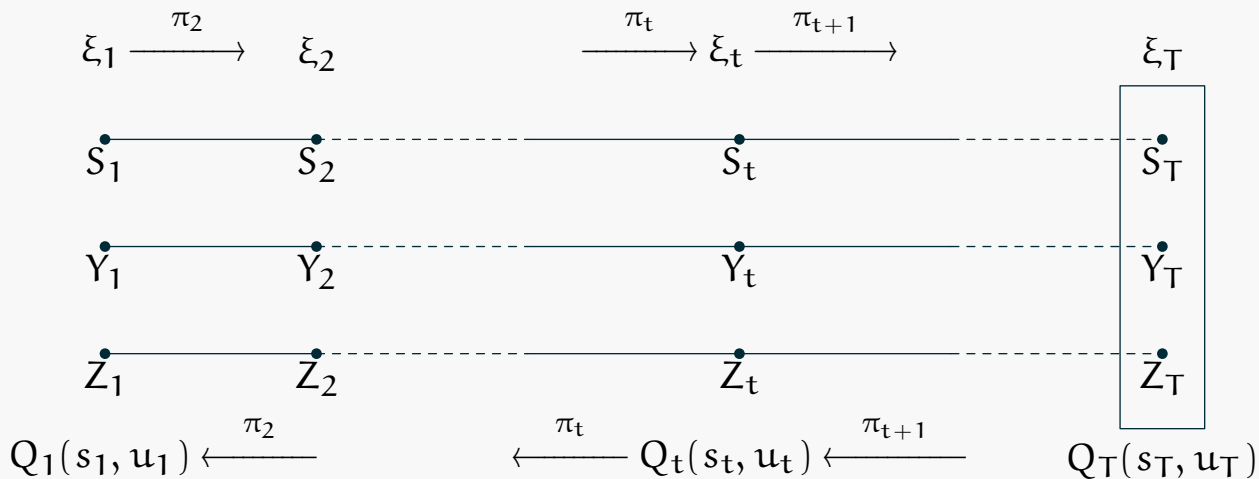
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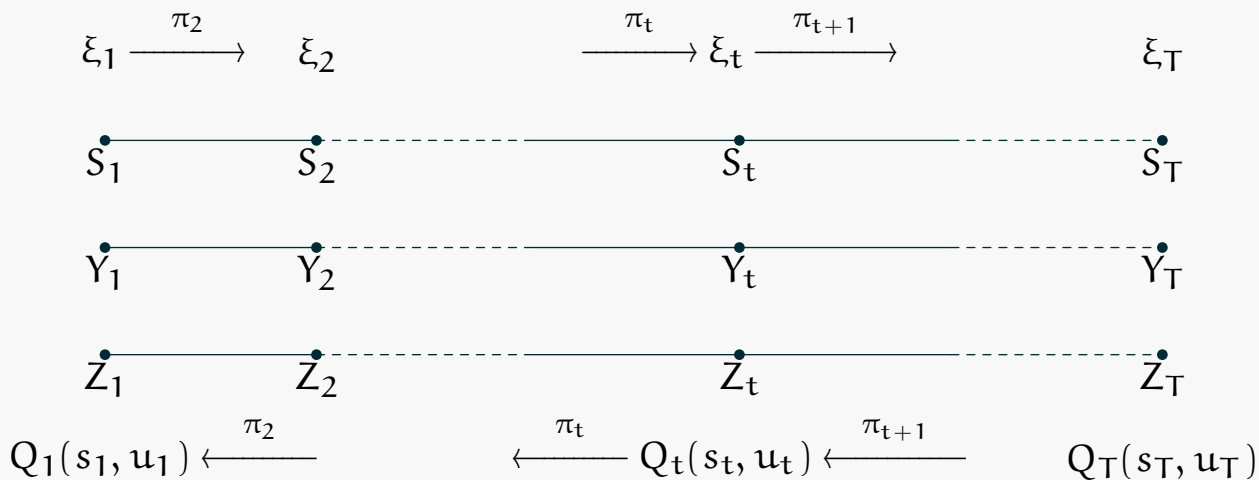
How to find team sequential equilibrium for POMDPs

Step 3: Averaging and optimization

▷ Q_t depends on the **future** policies $\pi_{t+1:T}$

▷ ξ_t depends on the **past** policies $\pi_{1:t-1}$

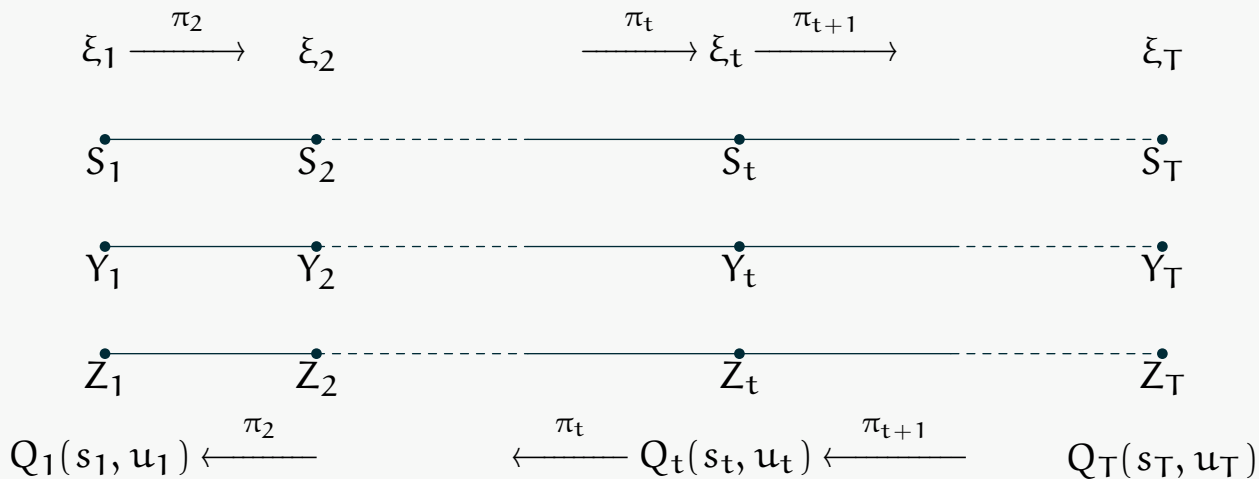
▷ **Averaged Q-function:** $\bar{Q}_t((y, z), u) = \sum_s \xi_t(s|y, z) Q_t(s, u)$



How to find team sequential equilibrium for POMDPs

Step 4: Local greedy update

$$\pi_t^{\text{next}} \in \mathcal{G}(\pi_{1:t-1}, \pi_{t+1:T}) := \arg \min_{a, z} \bar{Q}_t(y, z, a, z_+)$$



How to find a team sequential equilibrium for POMDPs

Improvement guarantee

For any $\pi = (\pi_1, \dots, \pi_T)$ and any t ,
let $\pi_t^{\text{next}} \in \mathcal{G}_t(\pi_{1:t-1}, \pi_{t+1:T})$. Then

$$J(\pi_{1:t-1}, \pi_t^{\text{next}}, \pi_{t+1:T}) \geq J(\pi_{1:t-1}, \pi_t, \pi_{t+1:T})$$

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Policy iteration algorithm

initialize $\pi^{(0)} = (\pi_1^{(0)}, \dots, \pi_T^{(0)})$ and $k \leftarrow 0$

repeat

$k \leftarrow k + 1$

 for $t \in \{T, \dots, 1\}$ do

$\pi_t^{(k)} \in \mathcal{G}_t(\pi_{1:t-1}^{(k-1)}, \pi_{t+1:T}^{(k)})$

 end for

until $\pi^{(k)} \equiv \pi^{(k-1)}$

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Convergence Guarantee

The PI algorithm converges to a
team sequential equilibrium, i.e.,

- ▶ Beliefs are consistent with policy
- ▶ Policy is BR to beliefs

Policy iteration algorithm

initialize $\pi^{(0)} = (\pi_1^{(0)}, \dots, \pi_T^{(0)})$ and $k \leftarrow 0$

repeat

$k \leftarrow k + 1$

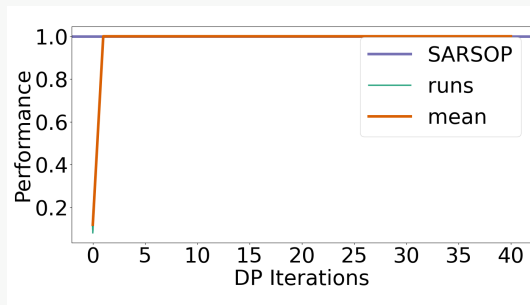
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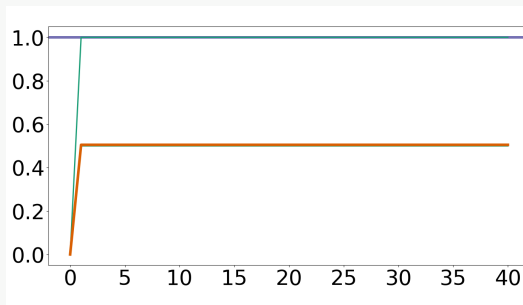
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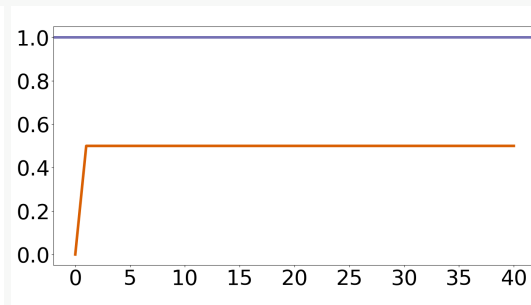
POMDP Experiments (with 1-bit memory)



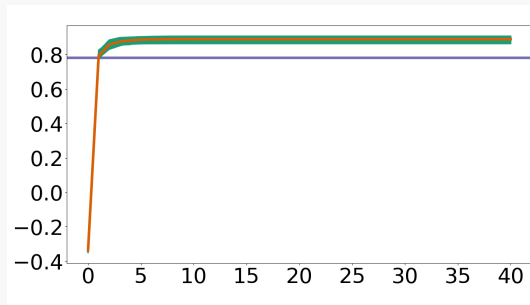
Cheesemaze



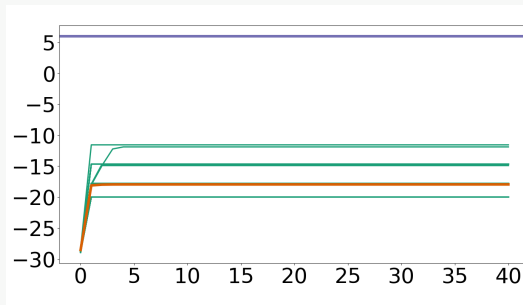
T-maze



Heaven Hell



Drone Surveillance



Shopping

- ▶ Agent-state with 1-bit memory
- ▶ Planning soln. No learning
- ▶ Different runs have different init
- ▶ Rest of the algo is deterministic

What's going wrong?

After the first iteration,
the policy is deterministic

Simple fix: Use conservatism

Conservative Policy Update

$$\mathcal{G}_t^\alpha(\pi_{1:t-1}, \pi_t, \pi_{t+1:T}) = (1 - \alpha)\pi_t + \alpha\mathcal{G}_t(\pi_{1:t-1}, \pi_{t+1:T}).$$

- ▶ In MDPs, conservatism is used to stabilize policy updates under approx policy evaluation
- ▶ We are dealing with exact policy evaluation here.
- ▶ But conservatism ensures that the policy remains stochastic

Simple fix: Use conservatism

Conservative Policy Update

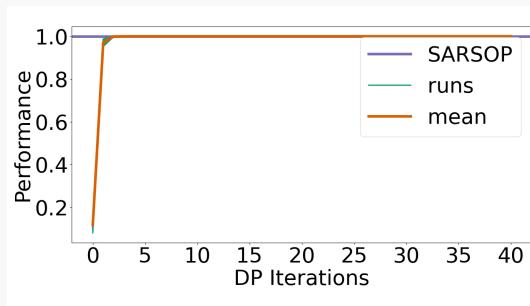
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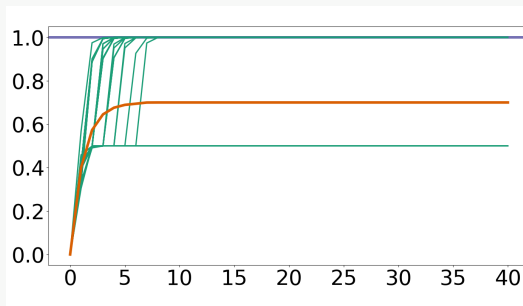
Choice of α

- ▶ α can change with iteration k
- ▶ Absence of conservatism: $\alpha^{(k)} \equiv 1$
- ▶ Constant conservatism: $\alpha^{(k)} \equiv \alpha$
- ▶ Policy averaging $\alpha^{(k)} = \frac{1}{k}$.

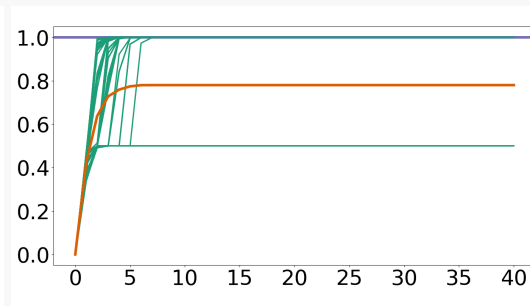
POMDP Experiments (with $\alpha=0.95$)



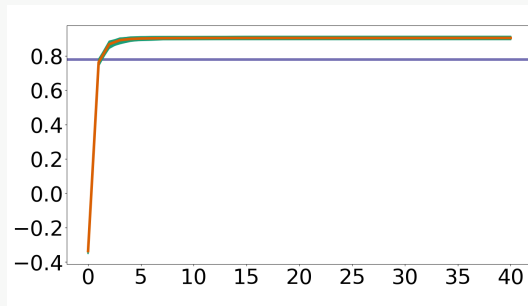
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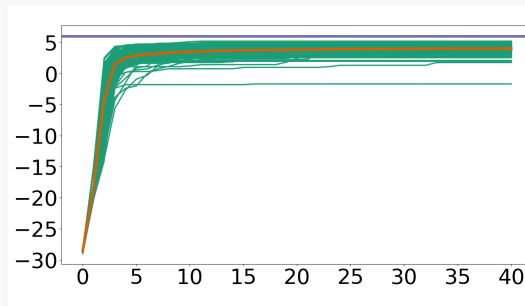
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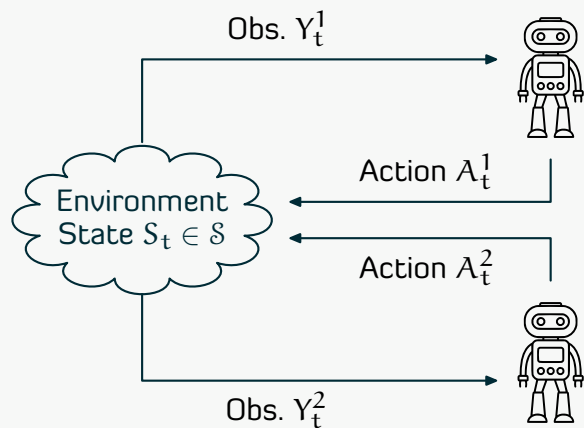


Shopping

Back to the multi-agent setting?

The same idea works!

Conservative Policy Iteration (CPI) for Dec-POMDPs



Basic idea

- ▶ Agents at time t , agents act in parallel
- ▶ Fix a total order in which the agents act, say $1 \rightarrow 2 \rightarrow 1 \rightarrow 2 \rightarrow 1 \rightarrow 2 \rightarrow \dots$
- ▶ Use the same algorithm

Dec-POMDP Experiments (1-bit memory, $\alpha=0.95$)

T	FB-HSVI	PF-MAA*	CPI
10	15.18	15.18	8.42
20	28.75	29.12	23.17
50	80.66	81.03	47.95
100	170.90	170.91	91.93

Tiger

T	FB-HSVI	PF-MAA*	CPI
10	26.31	26.31	18.02
20	52.13	49.37	35.80
50	128.95	122.56	89.37
100	249.92	234.06	178.60

Mars Rover

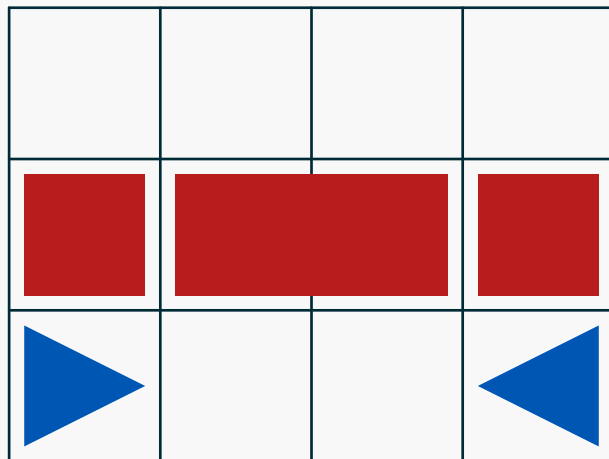
T	FB-HSVI	PF-MAA*	CPI
10	223.74	224.28	55.95
20	458.10	474.97	124.03
50	1134.70	1209.80	435.48
100	—	2433.51	771.35

Box Pushing

T	FB-HSVI	PF-MAA*	CPI
100	308.78	308.79	308.40
500	—	1539.56	1536.21
1000	—	3078.02	3074.56
2000	—	6154.94	6148.05

Recycling Robots

Why did it not work?



How to encourage coordination?

Bias the rewards towards cooperation

Biasing rewards towards cooperation

Risk-seeking policy evaluation ($\lambda > 0$)

$$Q_t^\pi(s, y, z, a, z_+) = r(s, a) + \frac{1}{\lambda} \log \left(\sum_{s_+, y_+} P(s_+, y_+ \mid s, a) \exp(\lambda Q_{t+1}^\pi(s_+, y_+, z_+, \pi_{t+1}(y_+, z_+))) \right)$$

► Rest of the algorithm remains the same!

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► Rest of the algorithm remains the same!

Improvement guarantee

For any $\pi = (\pi_1, \dots, \pi_T)$ and any t ,
let $\pi_t^{\text{next}} \in \mathcal{G}_t^\alpha(\pi_{1:t-1}, \pi_t, \pi_{t+1:T})$. Then

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RS-CPI

- For fixed λ , converges to risk-sensitive TSE
- Start with a large λ
- Slowly anneal λ with time

Dec-POMDP Experiments (with RS-CPI)

T	FB-HSVI	PF-MAA*	CPI	RS-CPI
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20	28.75	29.12	23.17	27.63
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100	170.90	170.91	91.93	120.30

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Recycling Robots

Conclusion

An algorithm to compute team
sequential equilibrium for Dec-POMDPs

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An algorithm to compute team sequential equilibrium for Dec-POMDPs

Key ideas

- ▶ For a fixed policy, $\{(S_t, Y_t, Z_t)\}_{t=1}^T$ is a MC.
- ▶ Easy to evaluate a policy $Q_t^\pi(s, \mathbf{a}, \mathbf{z}_+)$
- ▶ Easy to compute marginal distribution $\xi_t(s_t, y_t, z_t)$.
- ▶ Average wrt conditional belief $\xi_t(s_t \mid y_t, z_t)$

Conclusion

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Ongoing work

Generalize to

- ▶ Sampling based version
- ▶ Linear function approximation

Thank you



Amit Sinha



Matthieu
Geist



Jayakumar
Subramanian



Demos Teneketzis



IVADO



IDEaS
INNOVATION FOR DEFENCE
EXCELLENCE AND SECURITY

